Anas Alloush

[illegible]

Convolution :~

$$\underset{x(t)}{\overset{I/P}{}} \longrightarrow \boxed{h(t)} \longrightarrow \underset{y(t)}{\overset{O/P}{}}$$

$$y(t) = h(t) * x(t)$$

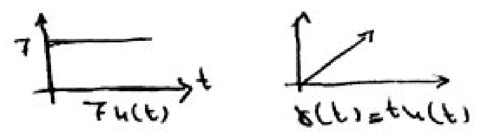
$$y(t) = \int_{-\infty}^{+\infty} h(\tau) \cdot x(t-\tau) \cdot d\tau$$

$$\text{oR} = \int_{-\infty}^{+\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

Note: 1- (t) in this integration is constant.
2- (τ) is a dummy Variable

Ex: Given a system Response $h(t) = \text{ramp}(t)$.
Find O/P $y(t)$

$$x(t) = 7u(t) \longrightarrow \boxed{h(t)} \longrightarrow \overset{?}{y(t)}$$



$$y(t) = \int_{-\infty}^{\infty} h(t) \cdot x(t-\tau).$$

$$= \int_{-\infty}^{+\infty} \tau \cdot u(t) \cdot 7u(t-\tau) \cdot d\tau$$

$\therefore$

$$= 7\int_{-\infty}^{+\infty} \tau \, dt$$

$$T\left(\frac{\tau^2}{2}\Big|_0^t\right) = T\frac{t^2}{2}\,u(t)$$

Ex: $e^{-t}u(t) \longrightarrow \boxed{e^{-2t}u(t)} \longrightarrow y(t)?$

$$y(t) = \int_{-\infty}^{+\infty} x(t-\tau)\,.\,h(\tau)\,d\tau$$

$$= \int_{-\infty}^{+\infty} e^{-(t-\tau)}u(t-\tau)\,.\,e^{-2\tau}u(\tau)\,d\tau$$

$$= \int_0^t e^{-(t-\tau)}\,.\,e^{-2\tau}\,d\tau$$

$$e^{-t}e^{+\tau}\,.\,e^{-2\tau}$$

$$= \int_0^t e^{-t}\,.\,e^{-\tau}\,d\tau$$

$$= -e^{-t}(e^{-\tau})\Big|_0^t = -e^{-t}[e^{-t} - \cancel{e^{-\tau}}\,1]$$

$$= [-e^{-2t} + e^{-t}]u(t)$$

System classification:-

in the new sheet!!

System Classification

① linear / non-linear

(★) when "superposition" applies → the system is <u>Linear</u>

Superposition :->

when input $\xrightarrow{\text{is}} x_1(t) \xrightarrow{\text{output is}} y_1(t) = x_1(t)$

when input $\xrightarrow{\text{is}} x_2(t) \xrightarrow{\text{output is}} y_2(t) = x_2(t)$

when input $\xrightarrow{\text{is}} x_3(t) = [x_1(t) + x_2(t)] \xrightarrow{\text{output}} y_3(t) = x_3(t)$

$\longrightarrow \quad = [x_1(t) + x_2(t)]$

$\longrightarrow \quad = y_1(t) + y_2(t)$

(★) when system is (<u>scalable</u>) it is $\boxed{\text{Linear}}$

<u>Ex</u> $y(t) = \cos[x(t-1)]$

$a\,y(t) \neq \cos[a\,x(t-1)] \longrightarrow \boxed{\text{non-Linear}}$

Linearity Examples :->

① $y(t) = x^2(t)$

* when input $= x_1(t) \longrightarrow$ output $= y_1(t) = x_1^2(t)$
* when input $= x_2(t) \longrightarrow$ output $= y_2(t) = x_2^2(t)$

Apply superposition :->

* when input $= x_3(t) = x_1(t) + x_2(t) \longrightarrow$ output $= y_3(t) = x_3^2(t)$

$y_3(t) = [x_1(t) + x_2(t)]^2 \Rightarrow$ اذا كان حاصل الخرج $\boxed{y_3(t)}$ يساوي حاصل جمع كل خرج منفرد مع الخرج الآخر ← $y_1 + y_2$ إذا يكون النظام خطي

$\boxed{\text{non-Linear}}$

$y_1(t) + y_2(t) \neq y_3(t)$

$x_1(t)^2 + x_2^2(t) \neq (x_1 + x_2)^2$

$\frac{dy}{dt} = ay(t) + x(t)$

input $\rightarrow x_1(t) \longrightarrow$ output $y'_1 = ay_1 + x_1$

input $\rightarrow x_2(t) \longrightarrow$ output $y'_2 = ay_2 + x_2$

input $\rightarrow x_3(t) = x_1(t) + x_2(t) \longrightarrow$ output $y'_3 = ay_3 + x_3$

Find the summation of $(y_1 + y_2)$. is it equal to the output of the system when its input is $x_3(t)$??

أوجد مجموع الخرج $(y_1 + y_2)$ هل المجموع يساوي الخرج عندما كان الدخل للنظام هو (x_3) ؟؟

$\boxed{y'_1 + y'_2 = a(y_1 + y_2) + (x_1 + x_2)} \underset{\text{يساوي}}{\Longleftrightarrow} \boxed{y'_3 = ay_3 + x_3}$

$\therefore$ The system is Linear

---

② Zero Input - Zero output (ZIZO)

All Linear systems $\longrightarrow$ ZIZO systems

NoT All ZIZO systems $\longrightarrow$ Linear systems

ZIZO :- is the system that have zero output $(y=0)$ when the input $(x=0)$

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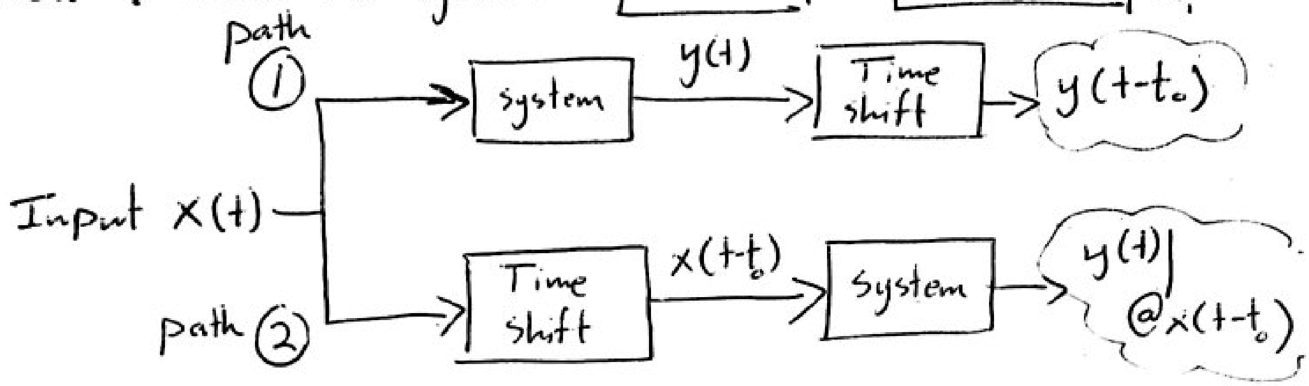
Time Variant / Invariant

the same Page (3)

* If a time shift in the input $x(t-t_0)$ cause the same time shift in the output $y(t-t_0)$ $\Rightarrow$ then the system is Time-Invariant

(إذا كانت الإزاحة في الزمن في الدخل تسبب نفس الإزاحة في الخرج
يكون النظام غير معتمد على الزمن Time-Invariant)

* How to Know the system is $\boxed{\text{Variant}}$ OR $\boxed{\text{Invariant}}$??



$\longrightarrow$ IF (1) = (2) The system is $\boxed{\text{Time-Invariant}}$

$\longrightarrow$ IF (1) $\neq$ (2) The system is $\boxed{\text{Time Variant}}$

Examples:-

(1) $y(t) = \cos(x(t))$

Apply path (1) $\overset{\text{system}}{\Longrightarrow}$ $y(t) = \cos(x(t))$ $\overset{\text{Time shift}}{\Longrightarrow}$ $y(t-t_0) = \cos(x(t-t_0))$

Apply path (2) $\overset{\text{Time shift}}{\Longrightarrow}$ $x(t-t_0)$ $\overset{\text{system}}{\Longrightarrow}$ $y(t)\big|_{\text{input @ } x(t-t_0)} = \cos(x(t-t_0))$

(1) = (2)

$\therefore$ System is Time-Invariant //

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$y(t) = x(t)\cos(t)$

Apply path (1) $\xrightarrow{\text{system}} y(t) = x(t)\cos(t) \xrightarrow{\text{Time shift}} y(t-t_0) = x(t-t_0)\cos(t-t_0)$

Apply path (2) $\xrightarrow{\text{Time shift}} x(t-t_0) \xrightarrow{\text{system}} y(t)\Big|_{\text{input @ } x(t-t_0)} = x(t-t_0)\cos(t)$

(1) $\neq$ (2) system Time-Variant

---

(4) Causal / non-Causal Systems

(*) Causal $\longrightarrow$ output depends on the $\boxed{\text{input in past } x(t-t_0) \quad \text{input in present } x(t)}$

output at time (t) does not depend on input at time greater than t

(*) non-Causal $\longrightarrow$ output depends on the $\boxed{\text{Future } x(t+t_0)}$

Examples :>

(1) $y(t) = x(t) \longrightarrow$ Causal

(2) $y(t) = x(t-t_0) \longrightarrow$ causal

(3) $y(t) = x(t+t_0) \longrightarrow$ non-Causal

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(5) memory / memoryless :>

عندما تعتمد قيمة خرج النظام $\boxed{y}$ على قيمة الدخل $\boxed{x}$ في زمن يقع بين $(t$ و $-\infty)$
يكون النظام له $\boxed{\text{ذاكرة} = \text{memory}}$

اذا كانت قيمة الخرج تعتمد فقط على قيمة الدخل في نفس اللحظة الزمنية (t)
يكون النظام $\boxed{\text{لا ذاكرة له} = \text{memoryless}}$

(*) $y(t) = \int_{-\infty}^{t} x(t)\,dt \longrightarrow$ memory system

(*) All $\boxed{\text{memoryless}}$ systems are $\boxed{\text{Causal}}$ systems

Examples

$y(t) = x(t-1) \rightarrow$ memory system

① $y(t) = e^{-3t} \cdot x(t+4) \longrightarrow$ memory system

لأن الخرج عند الزمن $\boxed{t}$ يعتمد على الدخل قبل 4 لحظات $\boxed{t+4}$

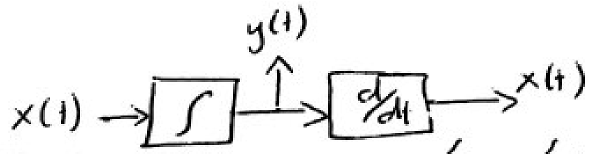
② $y(t) = 10 \cos\left(t\, x^2(t)\right) \longrightarrow$ memoryless system

لأن زمن الخرج يعتمد على زمن الدخل عند نفس اللحظة $\boxed{t}$

⑥ Invertible / Non-Invertible

* 2 different inputs gives 2 different outputs $\longrightarrow$ invertible system
* 2 different inputs gives the same output $\longrightarrow$ Non-Invertible system
* $\boxed{\text{Any periodic input} \longrightarrow \text{The system is Non-Invertible}}$

Examples :-

① $y(t) = \int_{-\infty}^{t} x(t)\, dt \Rightarrow$ 

يمكن عكس النظام إذا هو = $\boxed{\text{Invertible}}$

② $y(t) = \cos(x(t)) \Rightarrow \boxed{\text{Non-invertible system}}$

③ $y(t) = x^2(t) \Rightarrow \boxed{\text{Non-invertible}}$

$\because$ if input $= x(t) \rightarrow$ output $= y(t)$

$\because$ if input $= -x(t) \rightarrow$ output $= y(t)$

$\therefore$ 2 different inputs gives the same output.

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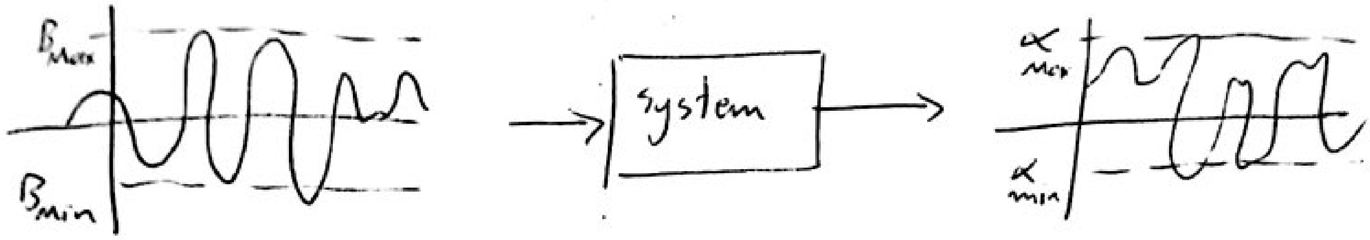
stable / unstable (BIBO) Page 6



محدود

* Bounded input should give Bounded output ⇒ stable

* If the output Amplitude goes to ∞ ⇒ unstable

[النظام المستقر هو الذي يكون الخرج فيه محدود ولا يساوي ∞]

Examples :-

(1) $y(t) = e^{x(t)}$

Replace $x(t)$ by a constant β → $e^{\beta} < \infty$

∴ system is BIBO $\boxed{\text{stable}}$

(2) $y(t) = \int_{-\infty}^{t} x(t)\,dt$

Replace $x(t)$ by a constant β ⇒ $\int_{-\infty}^{t} \beta \cdot dt$

⇒ $y(t) = \infty$ $\boxed{\text{unstable}}$

(3) $y(t) = 10 \cos[t x^2(t)]$

This output will always be bounded Regardless of $x(t)$ value

$(10 \cos[t x^2(t)] < 10)$

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System analysis:

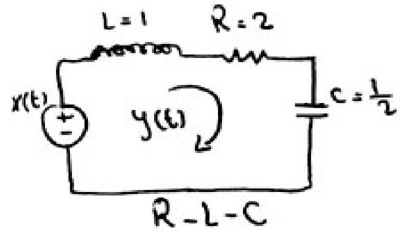
$$V_L(t) + V_R(t) + V_C(t) = x(t)$$

induct resis cap

$$L\frac{dy(t)}{dt} + Ry(t) + \frac{1}{C}\int_{-\infty}^{t} y(t)\,dt = x(t)$$

$$\frac{dy(t)}{dt} + 3y(t) + \frac{1}{1/2}\int_{-\infty}^{t} y(t)\,d(t) = x(t)$$

$$\frac{dy^2(t)}{dt^2} + \frac{3dy(t)}{dt} + 2y(t) = x(t)$$

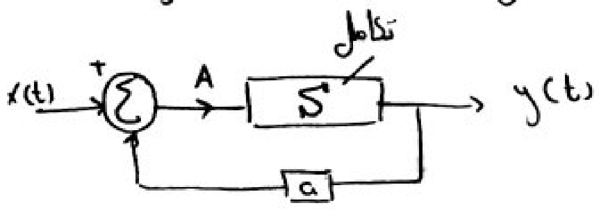


$$D^2y(t) + 3Dy(t) + 2y(t) = x(t)$$

$$y(t)\,[D^2 + 3D + 2] = x(t)$$

From Block diagram to differient equations

Ex1: what is the system Described by this Block Diagram



$$x(t) + ay(t) = A \quad , \quad A = \frac{d}{dt}y(t)$$

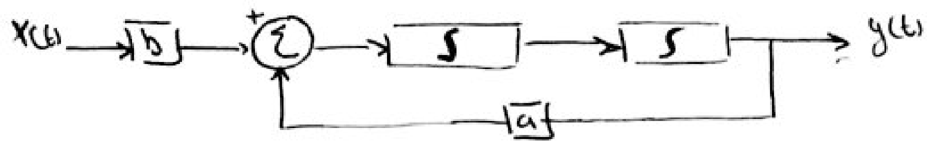
$$x(t) + ay(t) = \frac{dy(t)}{dt}$$

$$x(t) + ay(t) = Dy(t)$$

$x(t) = Dy(t) - ay(t)$

$x(t) = y(D - a)$

Ex 2:



$$bx(t) + ay(t) = \frac{d^2y(t)}{dt^2}$$

$$bx + ay = D^2y$$

$$bx = D^2y - ay$$

$$bx = y(D^2 - a)$$

From Different equation to Block Diagram Ex 1 Consider the system equation

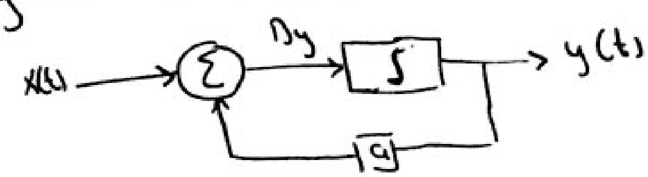
$$\frac{dy(t)}{dt} = ay(t) - x(t)$$

Draw the block Diagram

$Dy(t) = x(t) + ay(t)$

$Dy(t) = ay(t) = x(t)$

$y(D - a) = x(t)$



Total system response = Zero input Response + Zero state response

set input to Zero use intial conditions

1- set initial condition to zero
2- use input function

Zero input response:

[1] Real Roots

ZI solution

$$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + C_3^{\lambda_3 t} + \dots$$

Ex1: $\frac{d^2y(t)}{dt^2} + \frac{3dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt}$

Given I.C $\Rightarrow$ $y(t) = 0$

$y'(t) = -5$

[2] Repeated Real Roots

ZI solution

$$y(t) = C_{11} e^{\lambda_1 t} + C_{12} e^{\lambda_1 t} + t^2 C_{13} e^{\lambda_1 t} + \dots$$

Ex2: $D^2 y(t) + 6Dy(t) + 9y(t) = 3Dx(t) + 5x(t)$

Given I.C $\rightarrow$ $y(0) = 3$

$y(0) = -7$

[3] Complex Roots

ZI solution

$$y(t) = Ce^{\alpha t} \cos[\beta t + \theta]$$

Ex 3. $D^2y(t) + 4\,Dy(t) + 40\,y(t) = 0$

Given: $y(0) = 2$

$y'(0) = 16.78$

Solution: $D^2 + 4D + 40 = 0$

$\lambda^2 + 4\lambda + 40 = 0$

$(\lambda + 2 - j6)(\lambda + 2 + j6) = 0$

$\alpha = -2 \ , \ \beta = 6$

$y(t) = Ce^{\alpha t} \cos[\beta t + \theta]$

$y(t) = Ce^{-2t} \cos[6t + \theta] \rightarrow (1)$

using (I.C) to find C and θ

From IC

$y(0) = 2$

equation (1) Becomes:

$2 = Ce^{0} \cos[0 + \theta]$

$2 = C\cos\theta \rightarrow (2)$

using IC:

$y'(0) = 16.78$

$y'(t) = -6Ce^{-2t} \sin(6t + \theta) - 2Ce^{-2t} \cos(6t + \theta)$

Put every $t \to 0$

$$-16.78 = -6C \sin\theta - 2[2]$$

From equation (2)

$$16.78 = -6C \sin\theta = -3.463 \to (3)$$

$$\frac{\text{Equation 3}}{\text{Equation 2}} = \frac{C \sin\theta}{C \cos\theta} = \frac{-3.463}{2} = \tan\theta$$

$$\theta = \frac{-\pi}{3}$$

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Fourier Series :~ (F.S)

Trigonometric form :~

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$$

where ($\omega_0 = \frac{2\pi}{T_0}$) is called the fundamental Frequency

a_0 , a_n , b_n are the F.S coefficients.

① DC coefficient (a_0):

$$a_0 = \frac{1}{T_0} \int_0^{T_0} x(t) \, dt$$

② AC coefficient (a_n), (b_n).

$$a_n = \frac{2}{T_0} \int_0^{T_0} x(t) \cdot \cos(n\omega_0 t) \, dt$$

$$b_n = \frac{2}{T_0} \int_0^{T_0} x(t) \cdot \sin(n\omega_0 t) \, dt$$

F.S compact Form:

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

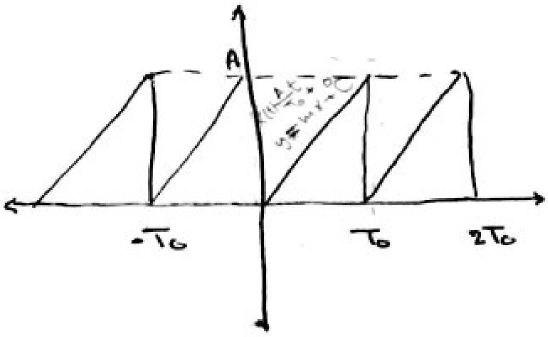
where :~

$C_0 = a_0$

$C_n = \sqrt{a_n^2 + b_n^2}$

$\theta = -\tan^{-1}\left(\frac{b_n}{a_n}\right)$

Ex: Find the Trigonometric form F.S of the given signal $x(t)$



Solution:

Given $x(t) = \frac{A}{T_0} t$

$$a_0 = \frac{1}{T_0} \int_0^{T_0} \frac{A}{T_0} t \, dt = \frac{A}{T_0^2} \left(\frac{t^2}{2} \Big|_0^{T_0} \right) = \frac{A}{2}$$

$$a_n = \frac{2}{T_0} \int_0^{T_0} x(t) \cdot \cos(n \omega_0 t) \cdot dt$$

$$a_n = \frac{2}{T_0} \int_0^{T_0} \frac{A}{T_0} t \cdot \cos(n \omega_0 t) \cdot dt$$

Recall

$\rightarrow \int x \cos x \, dx = \cos x + x \sin x$

$\rightarrow \int x \sin x \, dx = \sin x - x \cos x$

$\therefore$ Put $\cdots \rightarrow n\omega_0 t = u \rightarrow t = \frac{u}{n\omega_0}$

$\rightarrow n\omega_0 \, dt = du \rightarrow dt = \frac{du}{n\omega_0}$

$$a_n = \frac{2A}{T_0^2} \int_0^{T} t \cos(n \omega_0 t) \, dt$$

$$a_n = \frac{2A}{T_0^2} \int_0^{T_0} \frac{u}{n\omega_0} \cos[u] \left(\frac{du}{n\omega_0} \right)$$

$$a_n = \frac{2A}{T_0^2} \cdot \frac{1}{(n\omega_0)^2} \int_0^{T_0} u \cos u \, du$$

$$a_n = \frac{2A}{T_0^2 (n\omega_0)^2} \Big[\cos u + u \sin u \Big]_0^{T_0}$$

$$a_n = \frac{2A}{T_0^2 (n\omega_0)^2} \Big[\cos(n\omega_0 t) + (n\omega_0 t) \cdot \sin(n\omega_0 t) \Big]_0^{T_0}$$

$$a_n = \frac{2A}{T_0^2 (n\omega_0)^2} \Big(\big[\cos(n\omega_0 T_0) + (n\omega_0 T_0) \sin(n\omega_0 T_0)\big] - \big[\cos 0 + 0 \sin 0\big] \Big)$$

$$a_n = \frac{2A}{T_0^2 (n\omega_0)^2} \Big(\big[\cancelto{1}{\cos(n2\pi)} + (n2\pi) \cancelto{0}{\sin(n2\pi)}\big] - [1] \Big)$$

$$a_n = \frac{2A}{T_0^2 (n\omega_0)^2} (1 + 0 - 1) = 0$$

Note: $\rightarrow \cos 2\pi n = 1$

$\rightarrow \sin 2\pi n = 0$

for all n

* Recall:~

$$\int \sin(ax) = \frac{-1}{a} \cos(ax)$$

$$\int e^{\alpha x} dx = \frac{1}{\alpha} e^{\alpha x}$$

$$\sin(\pi t) = \cos(\pi t - 90)$$

لما نكون عندنا a_n ، b_n منحسب ال c_n

$\sqrt{a_n^2 + b_n^2}$ منوصل للسعة

$$\int_0^{T_0} \sin(n\omega_0 t) = 0 \quad \text{for all } n$$

$$\int_0^{T_0} \cos(n\omega_0 t) = 0 \quad \text{for all } n \quad \text{Except } (n=0)$$

$$\int_0^{T_0} \cos(m\omega_0 t)\sin(n\omega_0 t) = 0 \quad \text{for all } m, n$$

$$\int_0^{T_0} \sin(m\omega_0 t)\sin(n\omega_0 t)\,dt = \begin{cases} 0 \rightarrow \text{for all } (m \neq n) \\ T_0/2 \rightarrow \text{for all } (m = n) \end{cases}$$

*

$$b_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cdot\sin(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T_0}\int_0^{T} \frac{A}{T_0} t\cdot\sin(n\omega_0 t)\cdot dt$$

$$= \frac{2A}{T_0^2}\frac{1}{(n\omega_0)^2}\int_0^{T_0} u\sin(u)\,du$$

$$b_n = \frac{2A}{T_0^2(n\omega_0)^2}\left[\sin u - u\cos u\right]_0^{T_0}$$

$$b_n = \frac{2A}{T_0^2(n\omega_0)^2}\left[\sin(n\omega_0 t) - (n\omega_0 t)\cos(n\omega_0 t)\right]_0^{T_0}$$

$$b_n = \frac{2A}{T_0^2(n\omega_0)^2}\left[\sin(\cancel{n\omega_0 T_0})^{0} - n\omega_0 T_0\cos(\cancel{n\omega_0 T_0})^{1}\right] -$$

$$\left[\sin 0 - 0\cos 0\right]$$

$$b_n = \frac{2A}{T_0^2(n\omega_0)^2} \times (-n\omega_0 T_0) = \frac{-2A}{T_0 n\omega_0}$$

Date No

$$b_n = \frac{-A}{n\pi}$$

$$a_0 = \frac{A}{2}$$

$$b_n = \frac{-A}{n\pi}$$

$$a_n = 0$$

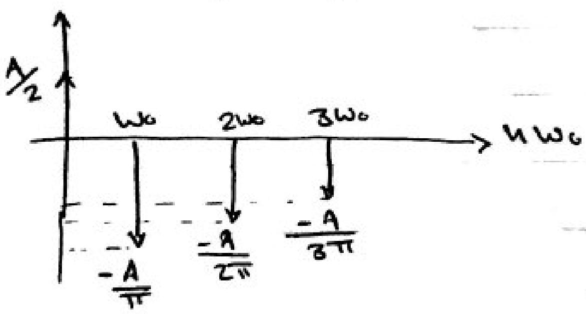
$$x(t) = a_0 + \sum [\cancel{a_n}^{\,0} \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$$

$$x(t) = \frac{A}{2} - \sum \frac{A}{n\pi} \sin(n\omega_0 t)$$

1st harmonic — 2nd harmonic — 3rd harmonic

$$x(t) = \frac{A}{2} - \frac{A}{\pi}\sin(1\omega_0 t) - \frac{A}{2\pi}\sin(2\omega_0 t) - \frac{A}{3\pi}\sin(3\omega_0 t) - \dots$$

Spectrum



لما يكون عندي a_n و b_n موجودين

$C_n = \sqrt{a_n^2 + b_n^2}$ [illegible]

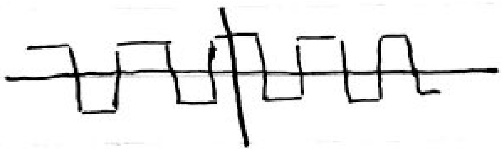
morning glory

F.S Coefficients for symmetrical signals:

① Zero mean symmetry

$a_0 = 0$

Ex:



② Odd symmetry

if $x(t)$ is odd

Then: $a_0 = 0$, $a_n = 0$ for all n values

$$b_n = \frac{4}{T_0} \int_0^{T_0/2} x(t) \cdot \sin(n w_0 t) \cdot dt$$

③ Even Symmetry

$b_n = 0$

$$a_0 = \frac{2}{T_0} \int_0^{T_0/2} x(t) \cdot dt$$

$$a_n = \frac{4}{T_0} \int_0^{T_0/2} x(t) \cos(n w_0 t) \cdot dt$$

④ Half-wave symmetry

Then: $a_0 = 0$ for all (n) values

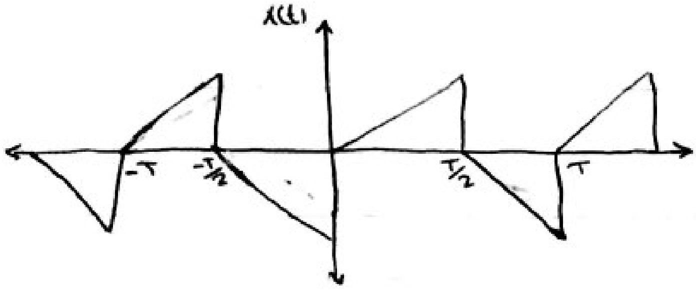
$a_n = 0$, $b_n = 0$ for (n) even values

but for (n) odd values :-

$$a_n = \frac{4}{T_0} \int_0^{T_0/2} x(t) \cdot \cos n\omega_0 t \, dt$$

$$b_n = \frac{4}{T_0} \int_0^{T_0/2} x(t) \cdot \sin n\omega_0 t \, dt$$

Ex:



⑤ Quarter-wave symmetry

A- Half-wave Symmet + Even wave symmetry

$$a_n = \frac{8}{T_0} \int_0^{T_0/4} x(t) \cdot \cos(n\omega_0 t) \, dt \rightarrow \text{only (n) odd values}$$

$a_0 = 0$, $b_n = 0$ $\rightarrow$ for all (n) values

$a_n = 0$ $\rightarrow$ for (n) even values

B- Half-wave symmetry + odd wave symmetry

$$b_n = \frac{8}{T_0} \int_0^{T_0/4} x(t) \cdot \sin(n\omega_0 t) \cdot dt \rightarrow \text{only for (n) odd values}$$

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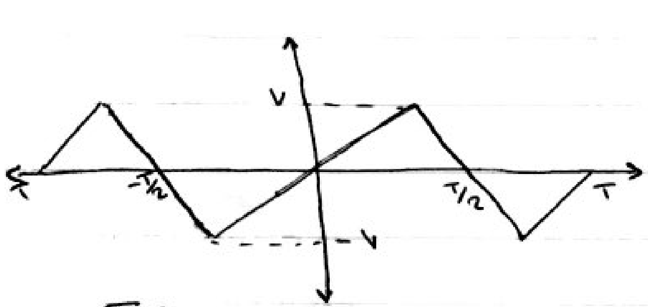
$a_0 = 0$, $a_n = 0$ → for all (n) values

$b_n = 0$ → for (n) even values

Ex: Find F.S.T.C for the given wave



$$b_n = \frac{8}{T_0} \int_0^{T_0/4} x(t) \cdot \sin(n\omega_0 t) \cdot dt$$

$$b_n = \frac{8}{T_0} \int_0^{T_0/4} \frac{4V}{T_0} t \cdot \sin(n\omega_0 t)\, dt$$

$$b_n = \frac{8 \times 4V}{T_0^2} \int_0^{T_0/4} t \cdot \sin(n\omega_0 t)\, dt \quad \text{—} (1)$$

$u = n\omega_0 t \rightarrow t = \frac{u}{n\omega_0}$

$du = n\omega_0 dt \rightarrow dt = \frac{du}{n\omega_0}$

Recall $\int u \sin u \, du = (\sin u - u \cos u)$

(1) Becomes:

$$b_n = \frac{8 \times 4V}{(n\omega_0)^2 T_0^2} \left[\sin u - u \cos u \right]_0^{T_0/4}$$

$$\frac{8\times 4v}{T_0^2(n\omega_0)}\left[\sin(n\omega_0 t) - n\omega_0 t\cos(n\omega_0 t)\right]_0^{T_0/4}$$

$$b_n = \frac{8\times 4v}{T_0^2(n\omega_0)}\left[\sin\left(n\omega_0 \frac{T_0}{4}\right) - \left[n\omega_0 \frac{T_0}{4}\right]\cos\left(n\omega_0 \frac{T_0}{4}\right)\right] - 0$$

$$b_n = \frac{8\times 4v}{T_0^2(n\omega_0)^2}\left[\sin\frac{\pi n}{2} - \frac{n\pi}{2}\cancel{\cos}\left(\frac{n\pi}{2}\right)\right]$$

$$b_n = \frac{8\times 4v}{T_0^2(n\omega_0)^2}\left[\sin\frac{\pi n}{2}\right]$$

F.S Exponential Form:

Recall:

$$\cos(n\omega_0 t) = \frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}$$

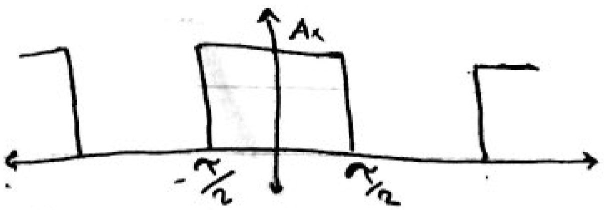
$$\sin(n\omega_0 t) = \frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}$$

F.S.

$$x(t) = \sum_{n=-\infty}^{+\infty} D_n \cdot e^{jn\omega_0 t}$$

where $\rightarrow$ $D_n = \frac{1}{T_0}\int_0^{T_0} x(t)\cdot e^{-jn\omega_0 t}$

Ex: Find Exponential Form F.S of given $x(t)$



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$$D_n = \frac{1}{T_0}\int_0^{T_0} x(t).\, e^{-jn\omega_0 t}$$

$$= \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} A.\, e^{-jn\omega_0 t} = \frac{A}{T_0}\cdot\frac{1}{-jn\omega_0}\left[e^{-jn\omega_0 t}\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{A}{T_0}\cdot\frac{-1}{jn\omega_0}\left[e^{-jn\omega_0 \tau/2} - e^{jn\omega_0 \tau/2}\right]$$

$$= \frac{A}{T_0}\cdot\frac{1}{jn\omega_0}\left[e^{jn\omega_0 \tau/2} - e^{-jn\omega_0 \tau/2}\right]$$

$$= \frac{A}{T_0}\cdot\frac{1}{n\omega_0}\left[\frac{e^{jn\omega_0 \tau/2} - e^{-jn\omega_0 \tau/2}}{j}\right] \longrightarrow \times \frac{2}{2}$$

$$= \frac{A}{T_0}\cdot\frac{1\times 2}{n\omega_0}\left[\sin(n\omega_0 \tfrac{\tau}{2})\right]$$

$$D_n = \frac{A}{T_0}\cdot\frac{2}{n(\frac{2\pi}{T_0})}\left[\sin(n\,\frac{2\pi}{T_0}\cdot\frac{\tau}{2})\right]$$

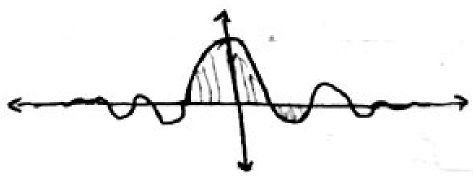
$$D_n = \frac{A}{T_0}\cdot\left[\frac{\sin(\frac{n\pi}{T_0}\cdot\tau)}{\frac{n\pi}{T_0}}\right] \longrightarrow \times \frac{\tau}{\tau}$$

$$D_n = \frac{A\tau}{T_0}\left[\frac{\sin(\frac{n\pi}{T_0}\cdot\tau)}{\frac{n\pi}{T_0}\cdot\tau}\right]$$

$$D_n = \frac{A\tau}{T_0}\cdot \operatorname{sinc}\left(\frac{n\tau}{T_0}\right)$$

Note:

$$\frac{\sin(x\pi)}{(x\pi)} = \operatorname{Sinc}(x)$$



Anas Allousн

Fourier Spectrum :-

$D_n = |D_n| e^{j\phi_n}$

$|D_n| \longrightarrow$ Amplitude

$\phi_n \longrightarrow$ Phase

Amplitude spectrum is always (Even)

Phase spectrum is always (odd)

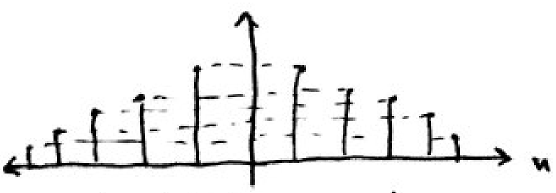
Ex: Given $D_n = \dfrac{0.3416}{0.2 + jn}$

Plot phase Amplitude spectrum

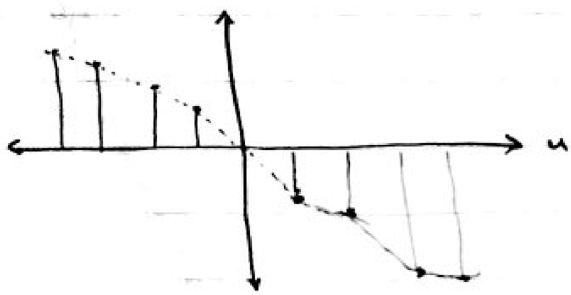
Solution:

$$|D_n| = \left| \frac{0.3416}{0.2+jn} \right| = \frac{\sqrt{0.3416^2}}{\sqrt{0.2^2 + n^2}} = \frac{0.3416}{\sqrt{0.04 + n^2}}$$

$$\phi_n = \frac{\angle 0.3416}{\angle 0.2+jn} = 0 - \tan^{-1}\left(\frac{n}{0.2}\right)$$



$|D_n|$ Amplitude spectrum



ϕ_n phase spectrum

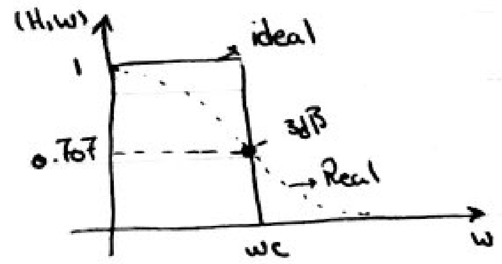
Finding power of $|D_n|$:

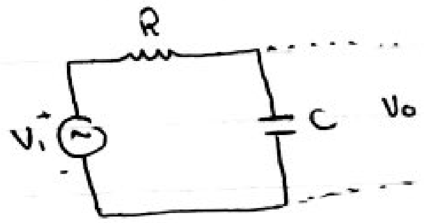
$$P = |D_0|^2 + 2\sum_{n=1}^{n=\infty} |D_n|^2$$

$$P = D^2\big|_{n=0} + 2D^2\big|_{n=1} + 2D^2\big|_{n=2} + 2D^2\big|_{n=3} + \dots$$

$$P = (1.708)^2 + 2(0.335)^2 + 2(0.17)^2 + 2(0.113)^2 + 2(0.085)^2 + \dots$$

Low pass Filter LPF





$$H(w) = \frac{V_o}{V_i} = \frac{1/jwC}{R + 1/jwC}$$

$$H(w) = \frac{1}{1 + jwRC}$$

↓

Transfer function of LPF

Wc is cutoff frequency

also called (Half-Power frequency) and (3 dB point).

3dB point is the point at which the magnitude drops to $\left(\frac{1}{\sqrt{2}}\right) = (0.707)$ of it's original value.

dB:

Filter response is often described in (dB)

$dB = 20 \log_{10}(\text{level}) \rightarrow$ voltage or current

$dB = 10 \log_{10}(\text{power})$

3dB point :-

$$H(\omega)\Big|_{\omega=\omega_c} = \frac{1}{\sqrt{2}} |H(\omega)|_{Maximum}$$

$$dB\left\{|H(\omega)|_{\omega=\omega_c}\right\} = dB\left\{\frac{1}{\sqrt{2}}|H(\omega)|_{Max}\right\}$$

$$= dB|H(\omega)|_{Max} + 20\log_{10}\left(\frac{1}{\sqrt{2}}\right)$$

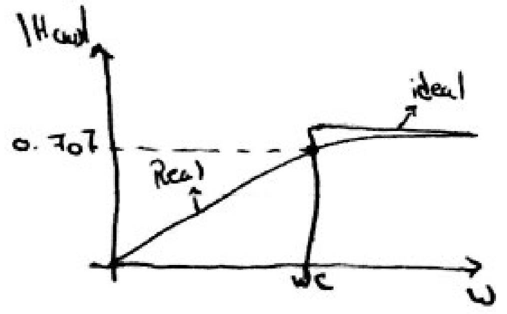
$$= dB|H(\omega)|_{Max} - 3$$

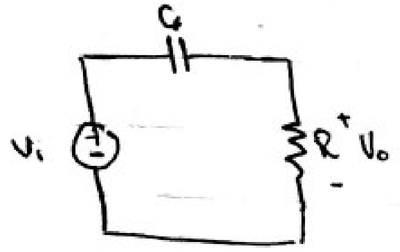
Note:

LPF will pass only low frequencies (lower than ω_c)

($\omega_c = \frac{1}{RC}$) using this equation we can design any LPF

High Pass Filter (HPF):





$$H(\omega) = \frac{V_o}{V_i} = \frac{R}{R + \frac{1}{j\omega C}}$$

$$H(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

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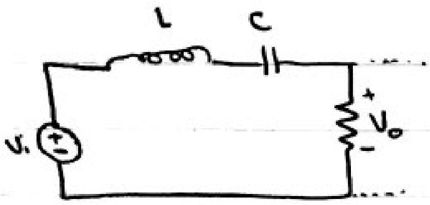
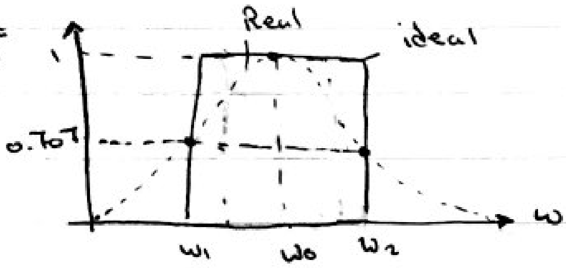
Band Pass Filter

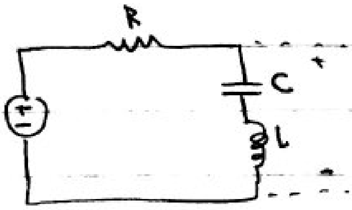


$$H(\omega) = \frac{V_o}{V_i} = \frac{R}{R + j(\omega L - \frac{1}{\omega C})}$$

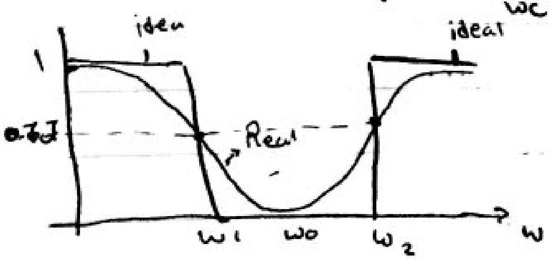
$$\omega_o = \frac{1}{\sqrt{LC}}$$



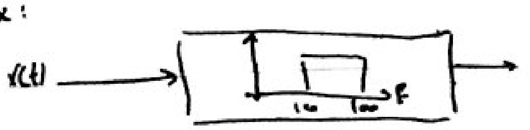
Band stop Filter



$$H(\omega) = \frac{V_o}{V_i} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$



Ex:



$= 50 \cos 50\pi t$

$x(t) = \cancel{10} + 30 \sin 10\pi t + 50 \cos 50\pi t$

$f = 5$ $\qquad$ $f = 25$

Test 2 2014 - 5 - 15

Q1. Given systems $S_1 - S_2 - S_3$ are connected as shown

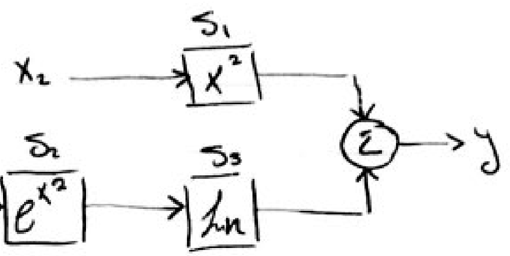
Consider $\begin{cases} x_1 = \cos 2\pi t + \sin 2\pi t \\ x_2 = \sin 2\pi t \end{cases}$

a. Check S_1, S_2 linearity

b. Check if S_2 is ZIZO / BIBO and prove your answer

c. Find system output $y(t)$.

d. What is the inverse system of S_1



a. $\boxed{S_1}$ $x_1 \to y_1 = x_1^2$ — (1)

$x_2 \to y_2 = x_2^2$ — (2)

$x_3 \to y_3 = x_3^2$ — (3)

$y_1 + y_2 \neq y_3 \to$ (non linear systems)

$\boxed{S_2}$ $x_1 \to y_1 = e^{x_1^2}$ — (1)

$x_2 \to y_2 = e^{x_2^2}$ — (2)

$x_3 \to y_3 = e^{x_3^2} = e^{(x_1 + x_2)^2}$ — (3)

$y_1 + y_2 \neq y_3 \to$ (non linear)

b. S_2 is not ZIZO

$e^0 = 1$ not ZIZO

S_2 is BIBO

$e^{B^2} \neq \infty \to B \neq \infty$ stable

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c. $x_2 \xrightarrow{S_1} \sin^2 2\pi t$

$x_1 \xrightarrow{S_2} e^{[\cos 2\pi t + \sin 2\pi t]^2}$

$x_1 \xrightarrow{S_3} \cos 2\pi t + \sin 2\pi t^2$

Recall

$\ln \infty = \infty$

$\ln e^x = x$

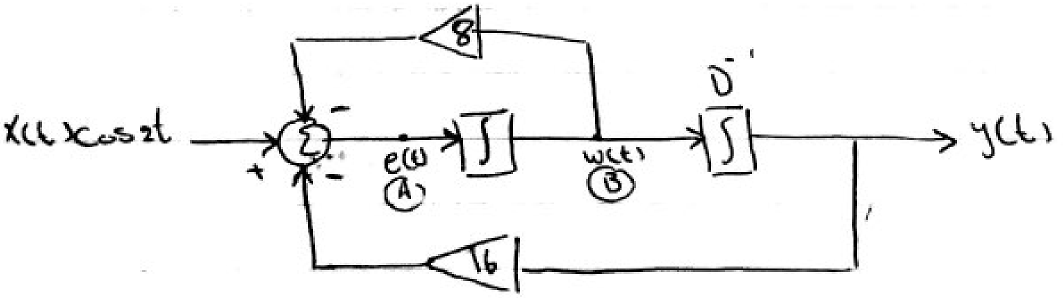
$\ln e^0 = 1$

$y \sin^2 2\pi t + \cos^2 2\pi t + \sin^2 2\pi t + 2\cos 2\pi t \sin 2\pi t$

d. not inverse because it gives use same an... wh... two different inputs.

Q2. b. Find Zero-Input Response for given system.

Initial conditions are $\boxed{\begin{matrix} y(0) = 2 \\ y'(0) = 6 \end{matrix}}$



$(A) = -16\, y(t) + x(t)\cos 2t - (B)\, 8$

$(B) = \frac{d\, y(t)}{dt} = D y(t)$

$(A) = \frac{d^2 y(t)}{dt} = D^2 y(t)$

$D^2 y(t) = -16 y(t) + x(t)\cos 2t - 8 D y(t)$

$D^2 y(t) + 8 D y(t) + 16 y(t) = \cancel{x(t)}^{\,0} \cos 2t$

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$\lambda^2 + 8\lambda + 16 = 0$

$(\lambda + 4)(\lambda + 4) = 0$

$\lambda_1 = -4 \quad , \quad \lambda_2 = -4$

$y(t) = C_1 e^{\overset{-4}{\lambda} t} + t C_2 e^{\overset{-4}{\lambda_1} t}$

$2 = C_1 + 0 \quad — (1)$

$C_1 = 2$

$y'(t) = -4C_1 e^{-4t} + C_2 e^{-4t} - 4tC_2 e^{-4t}$

at $t = 0$

$y'(0) = 6 = -4C_1 + C_2 \quad — (2)$

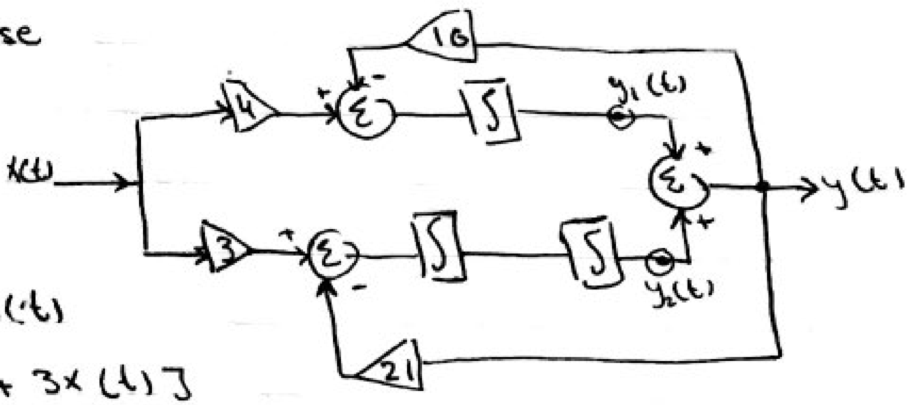
when $C_1 = 2$

$6 = -4(2) + C_2$

$C_2 = 14$

Q3. Find Z-Input Response

$I.C \longrightarrow \begin{bmatrix} y(0) = 0 \\ y'(0) = 2 \end{bmatrix}$



$y_1(t) = D^{-1}[10y(t) + 4x(t)]$

$y_2(t) = D^{-2}[-21y(t) + 3x(t)]$

$y(t) = y_1 + y_2$

$(\times D^2)\, y(t) = D^{-1}[4x(t) - 10y(t)] + D^{-2}[3x(t) - 21y(t)]$

$D^2 y(t) = D4x(t) - D10y(t) + 3x(t) - 21y(t)$

$D^2 y(t) + 10Dy(t) + 21y(t) = 4D\overset{\to 0}{\cancel{x(t)}} + \overset{\to 0}{\cancel{3x(t)}}$

$\lambda^2 + 10\lambda + 21 = 0$

$(\lambda_1 = -3) \quad (\lambda_2 = -7)$

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Qu. Consider system Equation :-

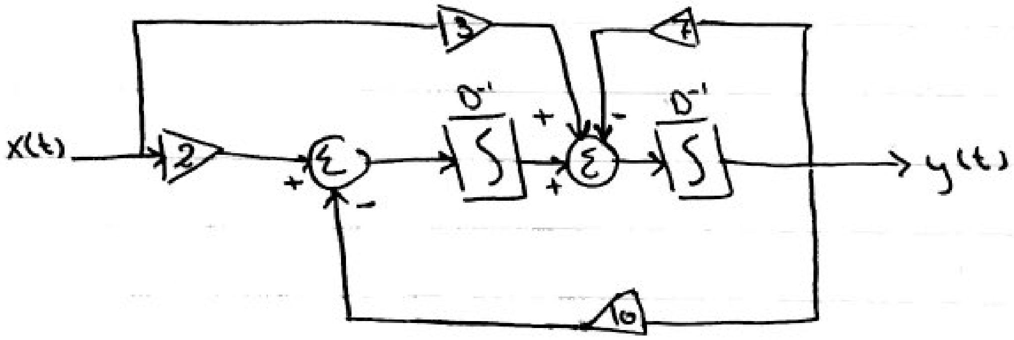
$$\frac{d^2 y(t)}{dt^2} + 7\frac{d}{dt}y(t) + 10\,y(t) = 3\frac{dx(t)}{dt} + 2x(t)$$

① Draw Block Diagram

② Find system Z-I solution? I.C → $\boxed{\begin{matrix} y(0) = 0 \\ y'(0) = 2 \end{matrix}}$

$(\times D^{2})$ $y(t) + 7D^{-1}y(t) + 10D^{-2}y(t) = 3D^{-1}x(t) + 2D^{-2}x(t)$

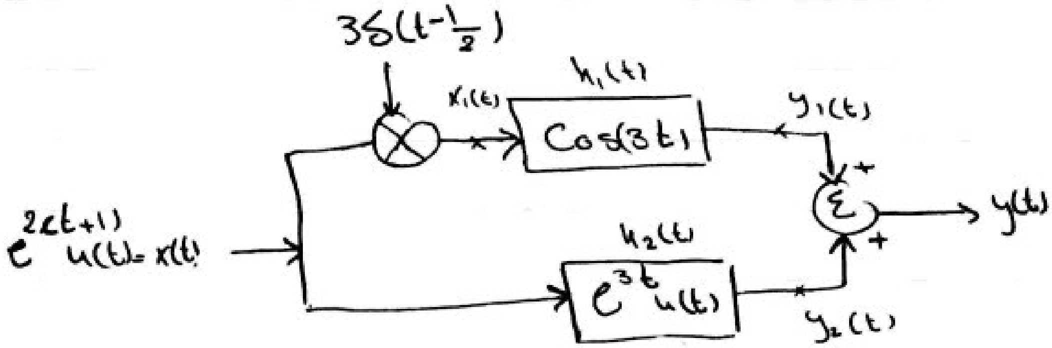
$y(t) = D^{-2}[-10y(t) + 2x(t)] + D^{-1}[-7y(t) + 3x(t)]$



Q.1 ① For $h_1(t)$ check linearity (prove) Test ② 12 - 5 - 2016

② For $h_2(t)$ check system stability (prove)

③ Find $y(t)$



$$\boxed{y_1(t) = x_1(t) * h_1(t)}$$

Finding $x_1(t)$

$$x_1(t) = e^{2(t+1)} u(t) \cdot 3\delta(t-\tfrac{1}{2})$$

$$x_1(t) = e^{2[\frac{1}{2}+1]} \cdot 3\delta(t-\tfrac{1}{2})$$

$$x_1(t) = 3e^{3}\delta(t-\tfrac{1}{2})$$

$$y_1(t) = \int_{-\infty}^{+\infty} \underbrace{3e^{3}\delta(\tau-\tfrac{1}{2})}_{x_1} \cdot \underbrace{\cos 3(t-\tau)}_{h_1} \cdot d\tau$$

$$y_1(t) = 3e^{3}\cos[3(t-\tfrac{1}{2})]$$

$$y_2(t) = x(t) * h_2(t)$$

$$= \int_{-\infty}^{\infty} e^{2(\tau+1)} u(t) \cdot e^{3(t-\tau)} u(t-\tau)\, d\tau$$

$$= \int_{0}^{t} e^{2(\tau+1)} \cdot e^{3(t-\tau)}\, d\tau$$

$$= \int_{0}^{t} e^{2\tau} \cdot e^{2} \cdot e^{3t} \cdot e^{-3\tau}\, d\tau$$

$$= e^{(2+3t)} \int_{0}^{t} e^{-\tau}\, d\tau$$

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$$= e^{-(2+3t)} \cdot \left[e^{-\tau} \Big|_0^{\tau} \right] = -e^{-2+3t} \cdot \left[e^{-t} - \cancel{e^{0}}^{\,1} \right] u(t)$$

$$y_2(t) = -e^{2+3t} \left[e^{-t} - 1 \right] u(t)$$

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QEx: Find F.S.T.C for this signal.

$$x(t) = 3 + \underbrace{\cos(4t + \tfrac{\pi}{4})}_{w_1 = 4} + \underbrace{\sin(10t + \tfrac{\pi}{3})}_{w_2 = 10}$$

$$W_1 = \frac{2\pi}{T_1} = 4 \longrightarrow T_1 = \frac{\pi}{2}$$

$$W_2 = \frac{2\pi}{T_2} = 10 \longrightarrow T_2 = \frac{\pi}{5}$$

$$\frac{T_1}{T_2} = \frac{5}{2} \longrightarrow T_0 = 5T_2 = 2T_1$$

$$T_0 = \pi$$

$$\therefore W_0 = \frac{2\pi}{T_0} = \frac{2\pi}{\pi} = 2 \longrightarrow \text{fundamental frequency}$$

Recall:

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\therefore x(t) = 3 + \cos(4t)\cos(\tfrac{\pi}{4}) - \sin(4t)\sin(\tfrac{\pi}{4}) + \sin(10t)\cos(\tfrac{\pi}{3}) + \cos(10t)\sin(\tfrac{\pi}{3})$$

$$x(t) = 3 + \frac{1}{\sqrt{2}}\cos(4t) - \frac{1}{\sqrt{2}}\sin(4t) + \frac{1}{2}\sin(10t) + \frac{\sqrt{3}}{2}\cos(10t)$$

$$\because F.S = a_0 + a_1\cos(nw_0t) + b_1\sin(nw_0t) + a_2\cos(nw_0t) + b_2\sin(nw_0t) + a_3\cos(nw_0t) + b_3\sin(nw_0t) + a_4\cos(nw_0t) + b_4\sin(nw_0t) + a_5\cos(nw_0t) + b_5\sin(nw_0t)$$

$a_0 = 3$ $\quad a_2 = \frac{1}{\sqrt{2}}$ $\quad a_4 = 0$

$a_1 = 0$ $\quad b_2 = -\frac{1}{\sqrt{2}}$ $\quad b_4 = 0$

$b_1 = 0$ $\quad a_3 = 0$ $\quad a_5 = \frac{\sqrt{3}}{2}$

$b_3 = 0$ $\quad b_5 = \frac{1}{2}$

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Q2 - Find Block diagram of this system then find it's Z-Input Response.

$$\frac{d^2y(t)}{dt^2} + \frac{7dy(t)}{dt} + 10y(t) = \frac{3dx(t)}{dt} + 2x(t)$$

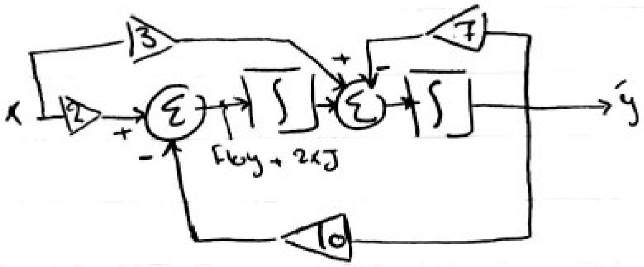
$(\times D^{-2})$; $D^2y + 7Dy + 10y = 3Dx + 2x$

$$y + 7D^{-1}y + 10D^{-2}y = 3D^{-1}x + 2D^{-2}x$$

$$y = D^{-2}[-10y + 2x] + D^{-1}[-7y + 3x]$$

$$\lambda^2 + 7\lambda + 10 = 0$$

$$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

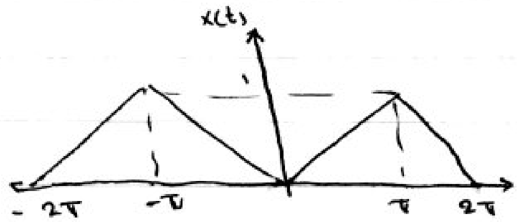


$b_n = 0$

$$a_0 = \frac{2}{T_0}\int_0^{T_0/2} x(t)\, dt$$

$$= \frac{2}{2\pi}\int_0^{\pi} \frac{1}{\pi} t \cdot dt = \frac{1}{\pi^2}\left(\frac{t^2}{2}\right)_0^{\pi}$$

$$a_0 = \frac{1}{2}$$



$T_0 = 2\pi$
$w_0 = 1$

$$a_n = \frac{4}{T_0}\int_0^{T_0/2} \frac{t}{\pi} \cos[n w_0 t]\, dt$$

$$a_n = \frac{4}{2\pi^2}\int_0^{T_0/2} t \cdot \cos(n w_0 t) \cdot dt = \begin{cases} 0 \longrightarrow \text{for (n) Even numbers} \\ \frac{-4}{n^2\pi^2} \longrightarrow \text{for (n) odd numbers} \end{cases}$$

$$a_n = \frac{2}{n^2\pi^2}\left[\cos(n\pi) - 1\right]$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n w_0 t)$$

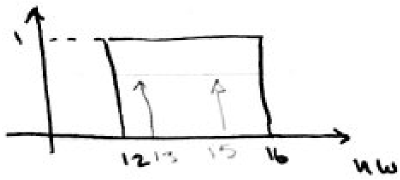
$$x(t) = \frac{1}{2} - \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{1}{n^2} \cos(n w_0 t)$$

Date No

$$x(t) = \frac{1}{2} - \frac{4}{\pi^2}\left[\cos(t) + \frac{1}{(3)^2}\cos(3t) + \frac{1}{(5)^2}\cos(5t) + \frac{1}{(7)^2}\cos(7t)\right.$$

$$\left. + \frac{1}{(9)^2}\cos(9t) + \dots\right]$$

* If $x(t)$ is the input of BPF what will be the output



$n = 13$, $n = 15$

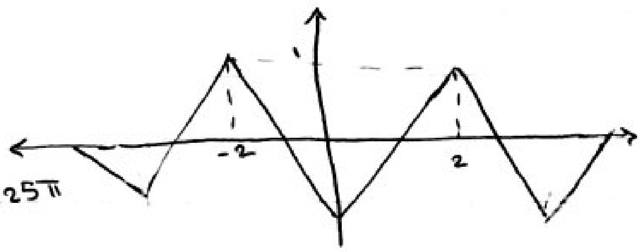
$$\frac{-4}{\pi^2}\left[\frac{1}{(13)^2}\cos(13t) + \frac{1}{(15)^2}\cos(15t)\right]$$

* Find the BPF output power

$$D_n = \frac{a_n - j\,b_n}{2}$$

$$P = 2\,\Sigma D_n$$

$$= 2\left[\frac{D_{n=13}^2}{2} + \frac{D_{n=15}}{2}\right]$$



If $x(t)$ is the input of BPF that has a center frequency $= 1.25\pi$ and Bandwidth $= \pi$.
Find the output of this BPF

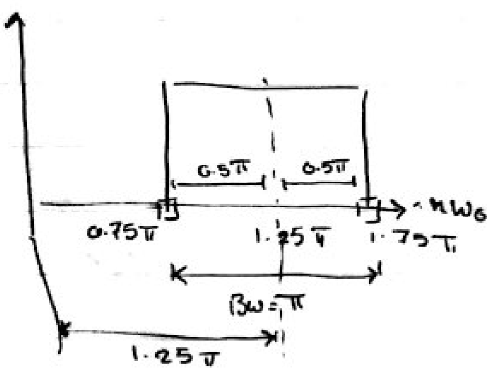
$T_0 = 4$

$n = 1 \rightarrow w_0 = 0.5\pi$

$$w_0 = \frac{2\pi}{T_0} = 0.5\pi$$

$(n = 3 \longrightarrow 3w_0 = 1.5\pi)$

Zero mean $a_0 = 0$



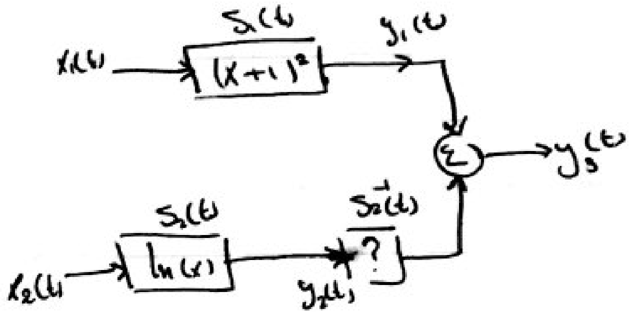
Anas Allousk

Q. For Given System $S_1(t)$, $S_2(t)$ are connected Test 2: 2013-12-26

as shown in

$x_1(t) = \cos 2\pi t$

$x_2(t) = |\sin 2\pi t|^2$ (where $\sin 2\pi t \neq 0$)



① check linearity of $S_2(t)$

② Is $S_2(t)$ ZIZO? BIBO? prove

③ Find system $S_2^{-1}(t)$ equation

④ Find $y_3(t)$

$x_1 \rightarrow \boxed{\ln} \rightarrow y_1 \ln x_1$

$x_2 \rightarrow \quad \rightarrow y_2 \ln x_2$

$x_3 \rightarrow \quad \rightarrow y_3 \ln x_3 = \ln(x_1 + x_2)$

$y_1 + y_2 \overset{?}{\neq} y_2$

non-linear

$y = \ln x$

$B_1 < |B| < \infty$

$y = \ln B \not< \infty$

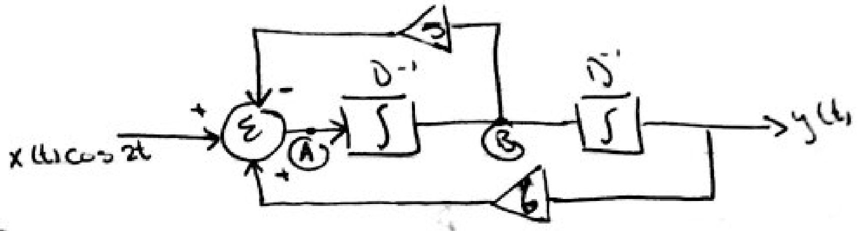
BIBO ✓ Because of the note

ZIZO ✗ Because of

$y_3(t) = [\cos 2\pi t + 1]^2 + \sin^2 \pi t$

$= \cos^2 2\pi t + 2\cos 2\pi t + 1^2 + \sin^2 2\pi t + 2 + 2\cos 2\pi t = y_3(t)$

$$B = \frac{d}{dt}y(t) = Dy(t)$$



I.C = $y(0) = 1$

$y'(0) = -4$

$$A = \frac{d^2y(t)}{dt^2} = D^2y(t)$$

$$A = 6y(t) + [x(t)\cos 2t] - 5B$$

$$D^2y(t) = 6y(t) = [x(t)\cos 2t] - 5Dy(t)$$

$$D^2y(t) + 5Dy(t) - 6y(t) = 6x(t)\,\overset{0}{\cancel{\cos}}\,2t$$

$$\lambda^2 + 5\lambda - 5 = 0$$

$$(\lambda - 1)(\lambda + 6)$$

$$\lambda_1 = 1, \ \lambda_2 = -6$$

Z I Response

$$y(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

$$= c_1 e^{t} + c_2 e^{-6t}$$

I.C#1 $y(0) = c_1 e^0 + c_2 e^0 = 1$

$$c_1 + c_2 = 1 \longrightarrow (1)$$

$$y'(t) = c_1 e^{t} - 6c_2 e^{-6t}$$

$$y'(0) = c_1 e^0 - 6c_2 = -4$$

$$c_1 - 6c_2 = -4 \longrightarrow (2)$$

Date No

Draw Block Diagram from this

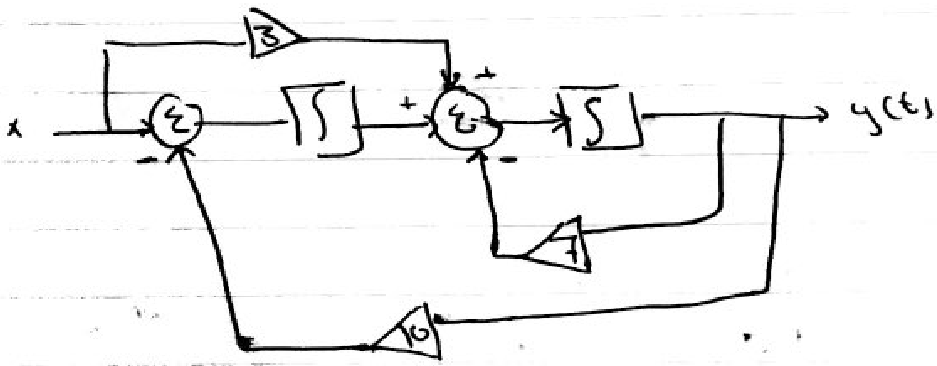
$D^2y + 7Dy + 10y = 3Dx + x$

$\frac{d^2}{dt^2} y(t) + 7\frac{d}{dt} y(t) + 10y(t) = 3\frac{d}{dt}x(t) + x(t)$

$D^{-2}(\; y + 7D^{-1}y + 10D^{-2}y = 3D^{-1}x + D^{-2}x$

$y = -7D^{-1}y - 10D^{-2}y + 3D^{-1}x + D^{-2}x$

$y = D^{-2}[x - 10y] + D^{-1}[3x - 7y]$



Date No

$y(t) = (t) * h(t)$

$= x_1(t) * h(t) + x_2(t) * h(t)$

$= \int_{-\infty}^{+\infty} \tau^2 e^{(t-\tau)} d\tau$

$= e^t \int_0^2 \tau^2 e^{-\tau} d\tau$

$n = 2 \quad a = -1$

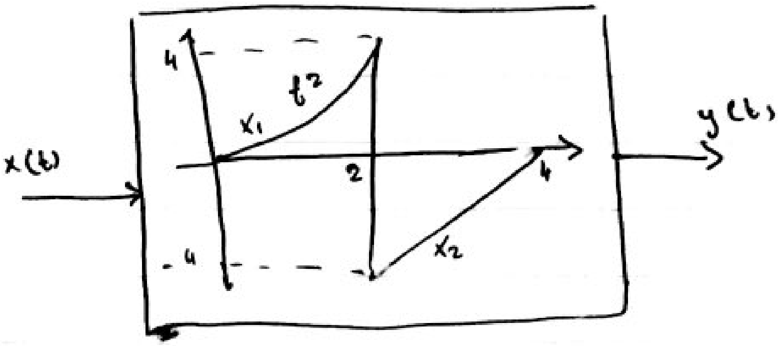
$= \frac{1}{-1} \tau^2 e^{-\tau} - \frac{2}{-1} \int_0^2 \tau^{2-1} e^{\tau} d\tau$

$= -\tau^2 e^{-\tau} + 2 \int_0^2 \tau^1 e^{-\tau} d\tau$

$= -\tau^2 e^{-\tau} + 2 \left[\frac{1}{-1} \tau e^{-\tau} - \frac{1}{-1} \int_0^2 \tau^{1-1} e^{-\tau} d\tau \right.$

$= -\tau^2 e^{-\tau} + 2 \left[-\tau e^{-\tau} - e^{-\tau} \right] \Big|_0^2$

$= (-10 e^{-2} + 4) e^t$



$x(t) = e^t u(t)$

Note:

$\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$

$h(t) = \begin{cases} t^2 & 0 < t < 2 \\ 2t - 8 & 2 < t < 4 \end{cases}$

TRIPOLI UNIVERSITY
FACULTY OF ENGINEERING
Electrical and Electronic Engineering Department

Subject: Systems and signals — Date: 15/12/2016
Course Code. : EE302 — Test 2: 20%
Fall 2016 — Time: 75 minutes

Note: write steps of solution, any direct result or multiple answers will not be considered

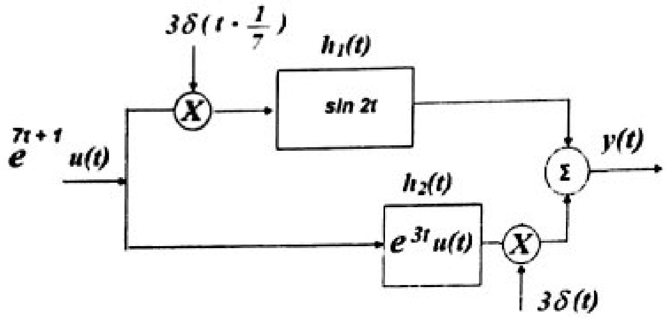
Q1)

A) Consider a given system input/output described by

$$y(t) = x(t)\cos(\omega t)\,u(t)$$

With answer prove; determine whether it is ***(a)*** memory less ***(b)*** causal ***(c)*** linear ***(d)*** time-invariant

B) – For given systems connected as shown in the figure below, using convolution property find $y(t)$



[14: 7 + 7]

Q2)

Input output system described by given differential equation :

$$\frac{d^2y(t)}{dt^2} + 10\,y(t) + 6\,\frac{dy(t)}{dt} = y(t) + x(t)\cos 2t$$

1- Draw system input output block diagram represent this system

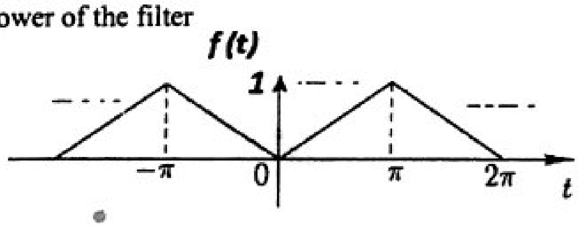
2- Dose system time varying or invariant, and why?

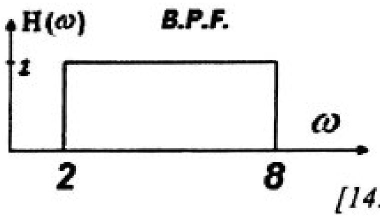
3- If I. C. Given, $y(0) = 2$, $\dot{y}(0) = 6$ Find Z. I. R. *[16: 7+2+7]*

Q3)

1) Find F.S.T.C. formula for periodic extension for given signal *f(t)*

2) If *f(t)* inserted to *B.P.F* given in figure below , sketch filter output spectrum and calculate the out power of the filter





[14: 7+7]

Hint:-

$$\int t\sin nt\,dt = \frac{1}{n^2}(\sin nt - nt\cos nt) + K$$

$$\int t\cos nt\,dt = \frac{1}{n^2}(\cos nt + nt\sin nt) + K$$

Good Luck

Y. Elsharef & K. Elgdamsi

302 13/5/2017

UNIVERSITY OF TRIPOLI
FACULTY OF ENGINEERING
Electrical and Electronic Engineering Department

Subject: Signals and Systems
Spring 2016
Student Name: -

Course: EE302
Final Exam: Theoretical [Marks: 20%]
Student No.:-

Date: 16–06 - 2016
Time: 20 minutes
Group:-

Complete /select / Answer the following question

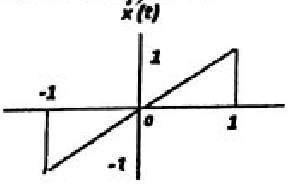
1 – All linear systems are ZIZO and all ZIZO systems are liner

True   False   Not necessary   only LTI systems

2- To convert signals representation from __Time__ domain to __Freq__ domain signals we use Fourier series for __Periodic signal__ and Fourier transform for __non-Periodic signal__.

3- Energy of Power signal is equal to ___∞___

4- Find the energy of the signal $6x(3-t)$



$$E_{x(t)} = 6^2 \int_{-1}^{1} x(t)\,dt = 36 \int_{-1}^{1} t^2\,dt = 36\left[\frac{t^3}{3}\right]_{-1}^{1} = 24J$$

5- Check the following system stability and causality

a. $h(t)=e^{-2t}u(t-1)$ __causal + stable__

b. $h(t)= \cos(10t)\, u(t+10)$ non Causal + stable

6- Impulse response of the system is:

Speed response   phase spectrum   not given   system identification

7- Determine the fundamental period if any of the signal $x(t) = 2\cos(10t+1) - \sin(4t-1)$

$$T_{o_1} = \frac{2\pi}{10} = \frac{\pi}{5}, \quad T_{o_2} = \frac{2\pi}{4} = \frac{\pi}{2} \quad \frac{T_{o_1}}{T_{o_2}} = \frac{2}{5} \text{ integer ration} \Rightarrow T_o = 2T_{o_1} = 5T_{o_2} = \pi$$

8- The output of an LTI system for any input signal $x(t)$ can be expressed as the convolution of the input signal with the system's impulse response.

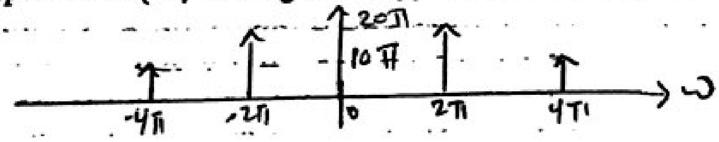
True   Falls   depend on signal type

9- Find the following

a- $\int_{-\infty}^{\infty}(t^2 + \cos \pi t)\delta(t)\,dt$ = ...1...

b- $\frac{d}{dt} r(t-5)$ = ...$u(t-5)$

10- Draw amplitude spectrum X (ω) for a signal: - $X(t) = 10 \cos 4\pi t + 20 \sin 2\pi t$



11- Modulation process is:

Time scaling   Amplitude scaling   power reduction   Frequency shifting

Good luck KE

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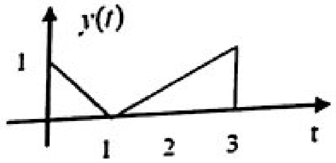
TRIPOLI UNIVERSITY
FACULTY OF ENGINEERING
Electrical and Electronic Engineering Department

Subject: Signals and Systems — Course: EE302 — Date: 16-06-2015
Spring 2015 — Final Exam: Problems [Marks: 80%] — Time: 110 minutes

Note: for all question write steps of solution, indicate used tables in your answer if any, any direct result will not be considered. All sketches must be labelled carefully. Any multiple answers will not be considered

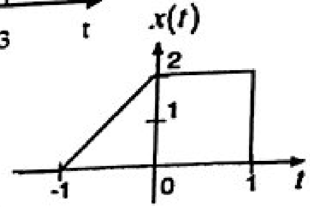
Q1 [25]

1- Describe given signal y(t) in terms of u(t) and r(t)



2- Compute $\int_{-\infty}^{\infty} y(t+1)\,\delta(t-1)dt$

3- For given signal *x(t)* sketch even and odd parts



4- Sketch 3*x(9-3t)*

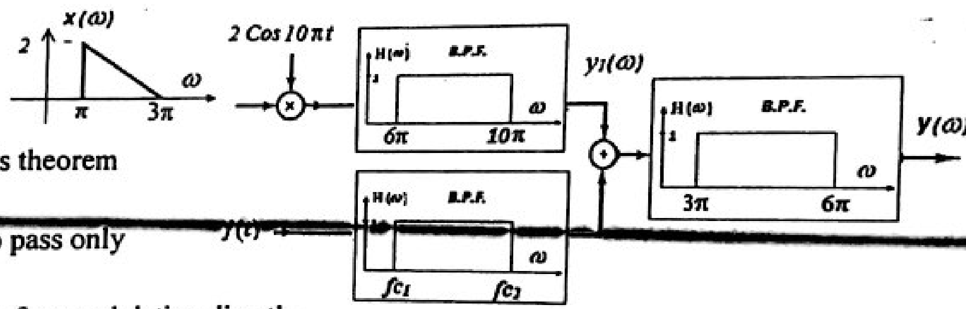
5- Find suitable measure of $f(t) = 5\,x(4t - 4)$

Hint: - for Q1, use : Amplitude scaling → Time scaling → Time shifting

Q2 [35]

For given C.T. Periodic signal *f(t)*, and *X(ω)* inserted to the system as shown in the figure below:-

Where $f(t) = \begin{cases} 1, & t \in [0,2); \\ -1, & t \in [2,4). \end{cases}$



a- Find *x(t)*

b- Find *X(ω)* energy using parsevals theorem

c- Find *f(t)* T.C. F.S formula

d- Design BPF fc_1 and fc_2 allowed to pass only fifth and seventh component

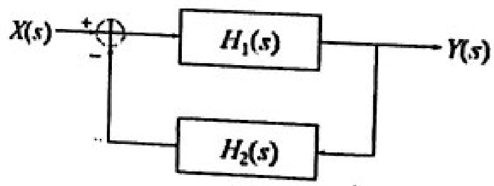
e- Find and sketch signal bandwidth after modulation directly

f- Sketch $y_1(\omega)$

g- Calculate overall system output power

Q3 [25]

1) Find Laplace transform for $x(t) = e^{-3t}\,(t + e^{4t})\cos 3t$



2) For given interconnected systems as shown in the figure

i) Find overall system transfer function " H(s)"

ii) Determine ***a*** and ***b*** such that the overall transfer function is:

$$H(s) = \frac{s}{(s+4)(s+5)}$$

where $H_1(s) = \dfrac{s}{(s+1)(s+a)}$ and $H_2(s) = \dfrac{b}{s}$.

iii) Find system output "*y(t)*" if $x(t) = u(t)$

iv) Define overall system output stability for $x(t) = e^{3t}u(t)$, prove your answer

Good luck R.E

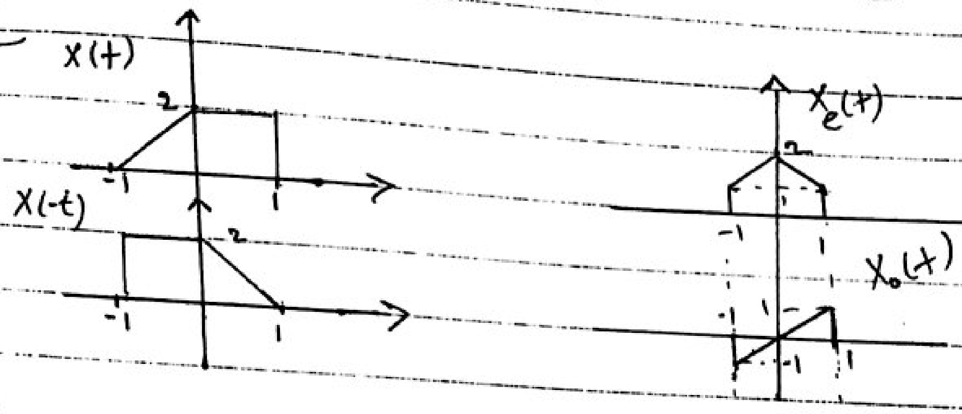
Spring 2016 Final Solution

Problems

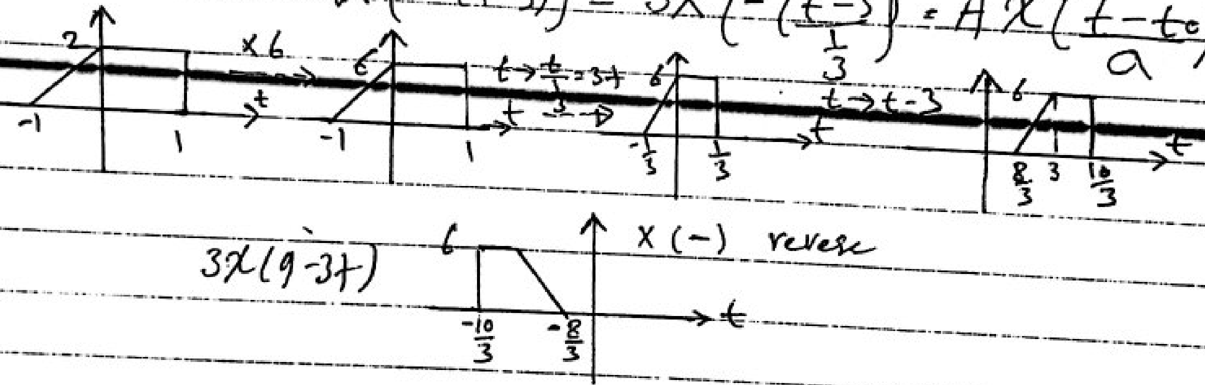
(1) $y(t) = u(t) - r(t) + \frac{3}{2} r(t-1) - \frac{1}{2} r(t-3) - u(t-3)$

(2) $= \int_{-\infty}^{\infty} \frac{t}{2}\Big|_{t=1} \delta(t-1)\,dt = \frac{1}{2}\int_{-\infty}^{\infty} \delta(t-1)\,dt = \frac{1}{2}$

(3)



(4) $3x(9-3t) = 3x(-3(t-3)) = 3x\left(-\left(\frac{t-3}{\frac{1}{3}}\right)\right) = Ax\left(\frac{t-t_0}{a}\right)$



(5) $5x(4t-4) = 5x(4(t-1)) = 5x\left(\frac{t-1}{\frac{1}{4}}\right)$

$E_{A(t)} = 5^2 \cdot \frac{1}{4} \cdot E_{x(t)} = \frac{25}{4}\int_{-1}^{1} |x^2(t)|\,dt$ $\quad x(t) = \begin{cases} 2(t+1) & \\ 1 & 0 < t \end{cases}$

$E_{x(t)} = \int_{-1}^{0} 4(t+1)^2\,dt + \int_{0}^{1} 4\,dt = 4\int_{-1}^{0}[t^2+2t+1] + 4 = \frac{16}{3}\,J$

$E_{A(t)} = \frac{25}{4} \cdot \frac{16}{3} = \frac{100}{3}\,J$

Q2) (a) $X(\omega) = 3 - \frac{\omega}{\pi}$ $\qquad \pi \le \omega \le 3\pi$

$$x(t) = (F.T)^{-1} X(\omega) = \frac{1}{2\pi}\int_{\pi}^{3\pi} X(\omega)\, e^{j\omega t}\, d\omega = \frac{1}{2\pi}\int_{\pi}^{3\pi}\left(3-\frac{\omega}{\pi}\right) e^{j\omega t}\, d\omega$$

8

$$= \frac{1}{2\pi}\left[\int_{\pi}^{3\pi} 3 e^{j\omega t}\, d\omega - \frac{1}{\pi}\int_{\pi}^{3\pi} \omega e^{j\omega t}\, d\omega\right]$$

$$= \frac{1}{2\pi}\frac{3}{jt}\left[e^{j\omega t}\right]_{\pi}^{3\pi} - \frac{1}{2\pi^2}\left[\frac{\omega e^{j\omega t}}{jt} - \frac{e^{j\omega t}}{(jt)^2}\right]_{\pi}^{3\pi}$$

$$x(t) = \frac{3}{2\pi jt}\left[e^{3j\pi t} - e^{j\pi t}\right] - \frac{1}{2\pi^2 jt}\left[3\pi e^{3j\pi t} - \pi e^{j\pi t}\right] + \frac{1}{2\pi^2 t^2}\left[e^{3j\pi t} - e^{j\pi t}\right]$$

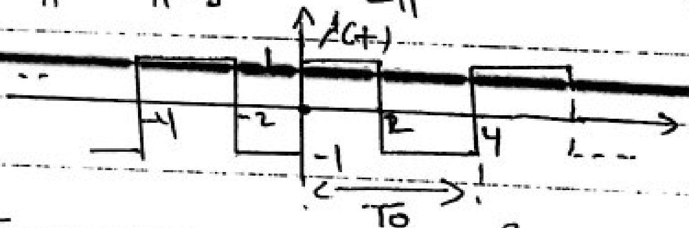
(b) $$E_{x(t)} = \frac{1}{2\pi}\int_{\pi}^{3\pi}|X(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{\pi}^{3\pi}\left[3-\frac{\omega}{\pi}\right]^2 d\omega$$

$$= \frac{1}{2\pi}\left[\int_{\pi}^{3\pi}\left[9 - \frac{6\omega}{\pi} + \frac{\omega^2}{\pi^2}\right]d\omega\right] = \frac{1}{2\pi}\left[9\omega - \frac{3\omega^2}{\pi} + \frac{\omega^3}{3\pi^2}\right]_{\pi}^{3\pi} = \boxed{\frac{4}{3}J}$$

(c) f(t) odd

$\Rightarrow a_n = 0$

$a_0 = 0$



$T_0 = 4$ s

$\omega_0 = \frac{\pi}{2}$ rad/s

$$b_n = \frac{2}{T_0}\int_0^{T_0} f(t)\sin n\omega_0 t\, dt = \frac{2}{4}\int_0^{2}\sin(n\omega_0 t)\, dt - \frac{2}{4}\int_2^{4}\sin(n\omega_0 t)\, dt$$

$$b_n = -\frac{1}{2}\left[\frac{-2}{n\pi}\cos\left(\frac{n\pi t}{2}\right)\right]_0^{2} - \frac{1}{2}\left[\frac{-2}{n\pi}\cos\left(\frac{n\pi t}{2}\right)\right]_2^{4}$$

$$b_n = \frac{1}{n\pi}\left[-\cos n\pi + \underbrace{\cos 0}_{1} + \underbrace{\cos 2n\pi}_{1} - \cos n\pi\right]$$

$$b_n = \frac{1}{n\pi}\left[2 - 2\cos n\pi\right]$$

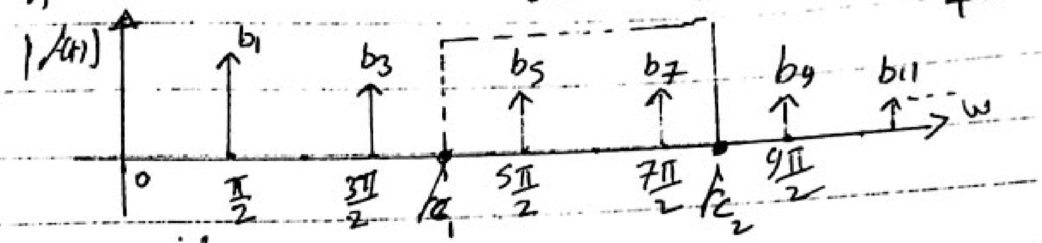
$$\boxed{f(t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi t}{2}}$$

2

fifth   Seventh

for n even $b_n = 0$

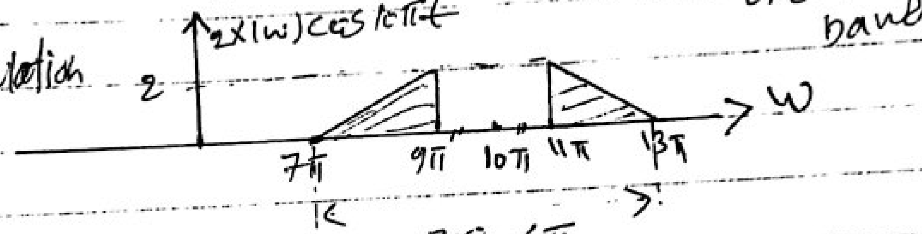
(d) $f(t) = \frac{4}{\pi}\sin\frac{\pi}{2}t + \frac{4}{3\pi}\sin\frac{3\pi}{2}t + \frac{4}{5\pi}\sin\frac{5\pi}{2}t + \frac{4}{7\pi}\sin\frac{7\pi}{2}t + \frac{4}{9\pi}\sin\frac{9\pi}{2}t + \cdots$



B.P.F $\frac{3\pi}{2} < 2\pi f_{c_1} \leq \frac{5\pi}{2}$ to guarantee pass

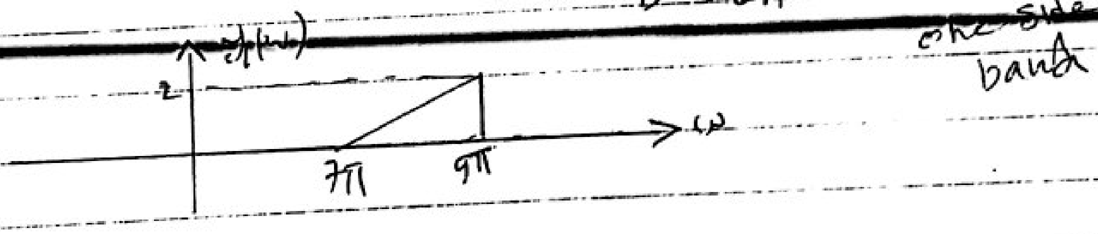
$\frac{7\pi}{2} \leq 2\pi f_{c_2} < \frac{9\pi}{2}$ "b_5, b_7"

(e) After modulation

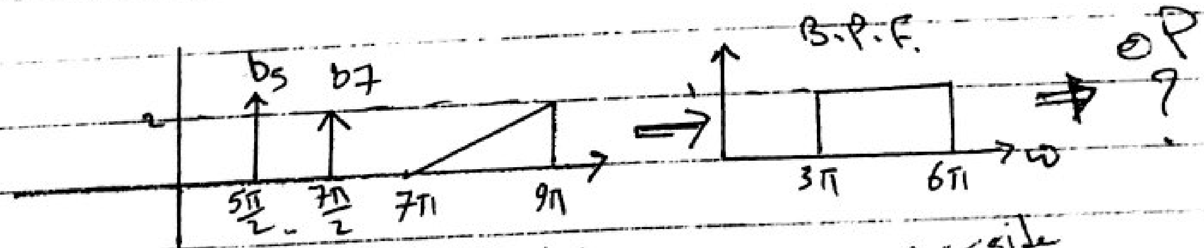


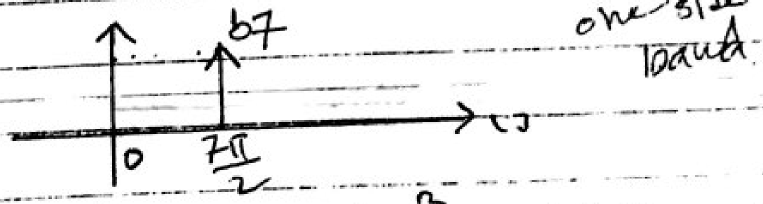
BW = 6π

(f)



(g)





$P(t)_{\text{output}} = 2|b_7|^2 = 2 \cdot \left|\frac{4}{7\pi}\right|^2 = \frac{16 \times 2}{49\pi^2}$ W

(3)

Q3) (1) $x(t) = te^{-3t}\cos 3t + e^{t}\cos 3t$

$$X(s) = -\frac{d}{ds}\left[\frac{s+3}{(s+3)^2+3^2}\right] + \frac{s-1}{(s-1)^2+3^2}$$

(2) [i] $H(s) = \dfrac{H_1(s)}{1+H_1(s)H_2(s)} = \dfrac{s}{(s+1)(s+a)+b}$

[ii] $\dfrac{s}{s^2+9s+20} = \dfrac{s}{s^2+(a+1)s+a+b} \Rightarrow a+1=9$, $a+b=20$

$\therefore\ a=8,\ b=12$

[iii] $H(s) = \dfrac{Y(s)}{X(s)} \Rightarrow Y(s) = X(s)H(s)$

$X(s) = \dfrac{1}{s} \Rightarrow Y(s) = \dfrac{1}{s}\cdot\dfrac{s}{(s+4)(s+5)} = \dfrac{1}{(s+4)(s+5)}$

$Y(s) = \dfrac{1}{(s+4)(s+5)} = \dfrac{A}{s+4} + \dfrac{B}{s+5}$

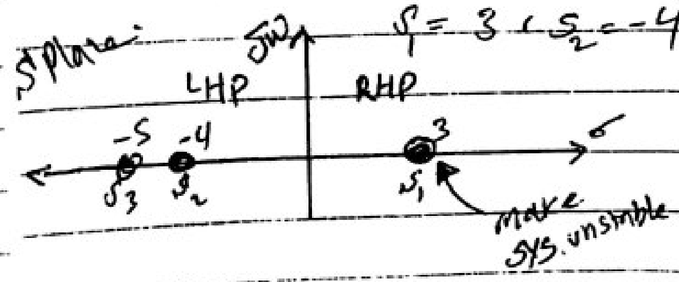
$A = \left.\dfrac{1}{s+5}\right|_{s=-4} = 1$, $B = \left.\dfrac{1}{s+4}\right|_{s=-5} = -1$, $Y(s) = \dfrac{1}{s+4} - \dfrac{1}{s+5}$

$y(t) = L^{-1}[Y(s)] = (e^{-4t} - e^{-5t})u(t)$

[iv] $X(s) = \dfrac{1}{s-3} \Rightarrow Y(s) = \dfrac{s}{(s-3)(s+4)(s+5)}$

Poles $(s-3)(s+4)(s+5) = 0$

$s_1 = 3$, $s_2 = -4$, $s_3 = -5$



System output is Not Stable

(4)

Test 2

UNIVERSITY OF TRIPOLI

FACULTY OF ENGINEERING

Electrical and Electronic Engineering Department

Subject: Signals and Systems
Spring 2016

Course Code: EE302

Marks: 20%

Date: 12 -05 - 2016

Time: 75 minutes

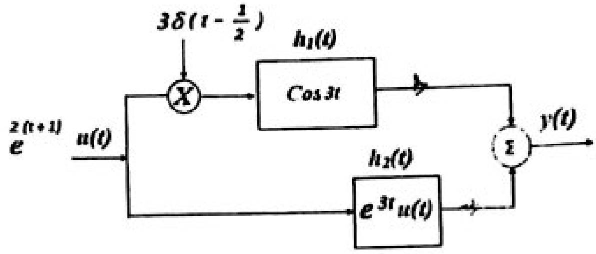
Note: *for all question write steps of solution, Indicate used tables in your answer if any, any direct result will not be considered. All sketches must be labelled carefully. Any multiple answers will not be considered*

Q1 [16: 3, 3, 8]

1- For $h_1(t)$,Check linearity, prove your answer

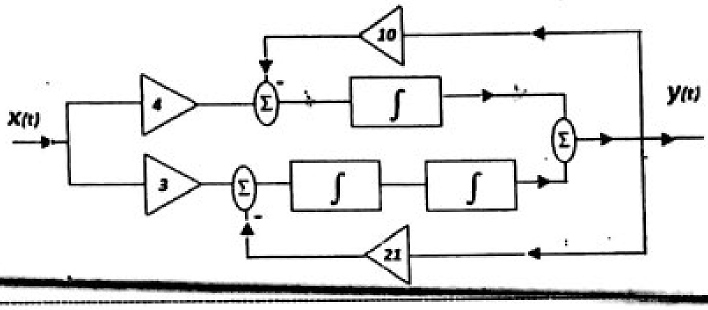
2- For $h_2(t)$ Check system stability, prove your answer

3- Find $y(t)$



Q2 [12] For LTI system shown in the figure below, find Zero Input response solution, given initial condition :-

$y(0) = 0$, $\dot{y}(0) = 2$



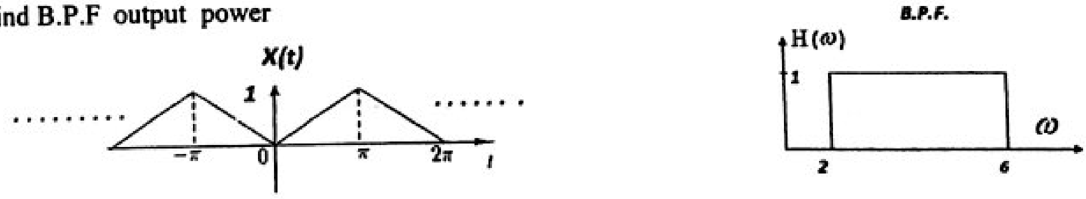
Q3 [18: 6, 6, 6]

For a given periodic signal $X(t)$ is inserted to the B.P.F " $H(\omega)$ " given below

1) Find X (t) T. F. S.C. formula

2) Sketch X (t) spectrum , phase and amplitude up to fifth component

3) Find B.P.F output power



[15]

$$\int t \sin at \, dt = \frac{1}{a^2}(\sin at - at \cos at) + K$$

$$\int t \cos at \, dt = \frac{1}{a^2}(\cos at + at \sin at) + K$$

Good luck

EE302 Test #2

spring 2016

Q1) (1) for $h_1(t) = \cos 3t$

$t \to t_1 \to \cos 3t_1$

$t \to t_2 \to \cos 3t_2$

$t \to t_3 = t_1 + t_2 \to \cos 3(t_1 + t_2)$

• it's clear $\cos 3t_1 + \cos 3t_2 \neq \cos 3(t_1+t_2)$ ∴ N.L.

(2) for $h_2(t) = e^{3t} u(t)$

as $t \to \infty$, $|h_2(t)| \to \infty$ ∴ Not stable

(3) $y(t) = e^{2(t+1)} * 3\delta(t-\tfrac{1}{2}) * h_1(t) + e^{2(t+1)} u(t) * h_2(t)$

$$y(t) = 3e^{3} \underbrace{\delta(t-\tfrac{1}{2}) * h_1(t)}_{h_1(t-\frac{1}{2})} + \int_0^t e^{2(\tau+1)} e^{3(t-\tau)} d\tau$$

$$= 3e^{3} h_1(t-\tfrac{1}{2}) + e^{3t+2} \int_0^t e^{-\tau} d\tau$$

$$= 3e^{3} \cos(3(t-\tfrac{1}{2})) + e^{3t+2} \left[e^{-\tau}\right]_0^t =$$

$$y(t) = 3e^{3}\cos(3(t-\tfrac{1}{2})) - e^{2}\left[e^{2t} - e^{3t}\right]u(t)$$

Q2) $y(t) = D^{-1}[4x(t) - 10y(t)] + D^{-2}[3x(t) - 21y(t)]$ $\times D^2$

$D^2 y(t) = D\,4x(t) - D\,10y(t) + 3x(t) - 21y(t)$

$D^2 y(t) + 10Dy(t) + 21y(t) = 4Dx(t) + 3x(t)$

$$\Rightarrow \boxed{\frac{d^2y(t)}{dt^2} + 10\frac{dy(t)}{dt} + 21y(t) = 4\frac{dx(t)}{dt} + 3x(t)}$$

Z.I.S. $x(t) = 0$

$\lambda^2 + 10\lambda + 21 = 0$

$(\lambda+3)(\lambda+7) = 0 \Rightarrow \lambda_1 = -3,\ \lambda_2 = -5$

Z.I.R. solution $y(t) = c_1 e^{-3t} + c_2 e^{-7t}$

give $y(0) = 0 \Rightarrow c_1 + c_2 = 0$ — (1)

$\dot{y}(0) = 2 \qquad -3c_1 - 7c_2 = 2$ — (2)

(1) with (2)

$\Rightarrow c_1 = \frac{1}{2}, \; c_2 = -\frac{1}{2}$

$$\boxed{y(t) = \frac{1}{2}e^{-3t} + \frac{1}{2}e^{-7t}}$$

Q3) (1) $x(t)$ Even $\Rightarrow b_n = 0$ $\qquad -\frac{t}{\pi} + 2$

$w_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2\pi} = 1$

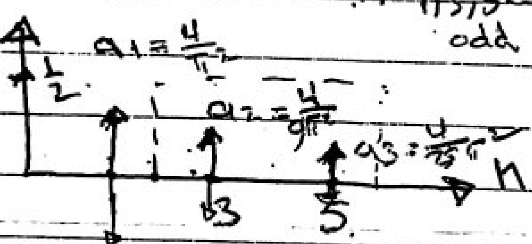
$$a_0 = \frac{2}{T_0}\int_0^{\pi} \frac{t}{\pi}\,dt = \frac{2}{2\pi}\left[\frac{t^2}{2}\right]_0^{\pi} = \boxed{\frac{1}{2} = a_0}$$

$$a_n = \frac{4}{2\pi}\int_0^{\pi} \frac{t}{\pi}\cos n w_0 t\,dt = \frac{-4}{n^2\pi^2}[\cos n\pi - 1]$$

$$x(t) = a_0 + \left(\frac{-4}{n^2\pi^2}\sum_{n=1,3,5}^{\infty} \cos nt\right)$$ where $w_0 = 1$

odd

(2)



for n even $a_n = 0$

$$x(t) = \frac{1}{2} - \frac{4}{\pi^2}\left[\cos t + \frac{1}{9}\cos 3t + \frac{1}{25}\cos 5t + \cdots\right]$$

(3) B.P.F pass only a_3, a_5

$$y(t) = \frac{4}{\pi^2}\cdot\frac{1}{9}\cos 3t + \frac{4}{25\pi^2}\cos 5t$$

$$\langle y(t)\rangle^2 = \left[\left(\frac{4}{9\pi^2}\right)^2 + \left(\frac{4}{25\pi^2}\right)^2\right]/2 =$$

TRIPOLI UNIVERSITY
FACULTY OF ENGINEERING
Electrical and Electronic Engineering Department

Subject: Systems and signals
Course Code. : EE302
Spring 2014

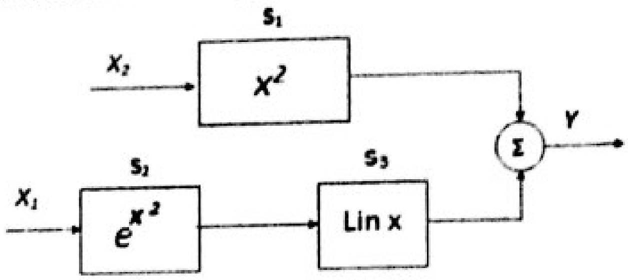
Date: 15/05/2014
Test 2
Time: 75 minutes

Note: write steps of solution, any direct result will not be considered.
: any multiple answers will not be considered

Q1) **– a given systems S_1 , S_2 and S_3 are connected as showen in the figure below, consider**

$x_1 = \cos 2\pi t + \sin 2\pi t$,

$x_2 = \sin 2\pi t$



a) Check linearity for S_1 and S_2

b) Check if S_2 ZIZO or BIBO, prove your answer?

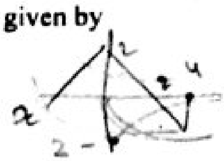
c) Find system output equation "y"

d) What is the inverse system of S_1

[20]

Q2)

a) **Consider impulse response for a system *h* (t) = - 2 e^{-t} u (t).** Find output of the system if input signal is given by



$$x(t) = \begin{cases} 2+t & -2 \le t \le 0 \\ 2-t & 0 < t \le 4 \\ 0 & \text{Other wise} \end{cases}$$

t	$-2e^{-t}\,u(t)$
0	-2
4	-0.036
-2	

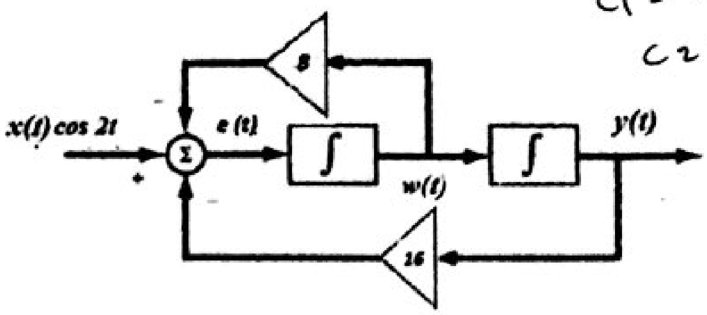
$0 = -2e^{-t}$

b) For continues time system shown in the figure below, find Zero Input response solution, given initial condition

$y(0) = 2 , \dot{y}(0) = 6$

$C_1 = 2$

$C_2 = 14$



$\lambda \times 4$

$\lambda \times 4$

$\lambda^2 + 4\lambda \times 4\lambda$

[20]

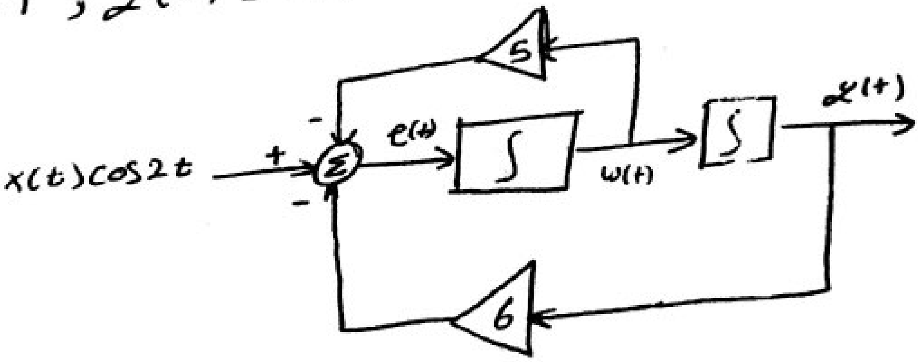
Good Luck

K.E.

Hint:- $\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$

t^t

Ex: For continues time system shown in the find Zero input responce solution, given initial condition: $y(0)=1$, $\dot{y}(0)=-4$



Solution

$$e(t) = -5\,w(t) - 6y(t) + x(t)\cos 2t$$

$\because$ $e(t) = \dfrac{d^2y(t)}{dt^2}$, $w(t) = \dfrac{dy(t)}{dt}$

$$\frac{d^2y(t)}{dt^2} = -5\frac{dy(t)}{dt} - 6y(t) + x(t)\cos 2t$$

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = x(t)\cos 2t$$

$\because$ $D = \dfrac{d}{dt}$ & for zero input solution $(x(t)=0)$

$\therefore$ $(D^2 + 5D + 6)\,y(t) = 0$

$(\lambda+2)(\lambda+3)=0 \Rightarrow \lambda_1 = -2$, $\lambda_2 = -3$

$y_0(t) = C_1 e^{-2t} + C_2 e^{-3t} \Longrightarrow ①$

$\because$ intial conditions $y(0)=1$ & $\dot{y}(0)=-4$

$y(0) = 1 = C_1 + C_2 \Rightarrow ②$

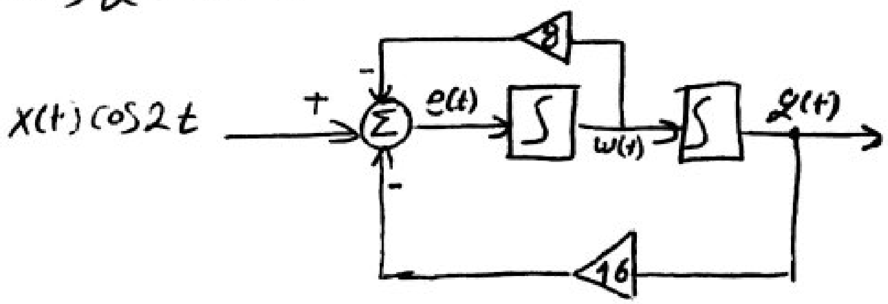
$\dot{y}_0(t) = -2C_1 e^{-2t} - 3C_2 e^{-3t}$

$\dot{y}_0(0) = -4 = -2C_1 - 3C_2 \Rightarrow ③$

② $\xrightarrow{\text{in}}$ ③ $\quad -4 = -2C_1 - 3(1-C_1)$

$-4 = -2C_1 - 3 + 3C_1 \Rightarrow \boxed{C_1 = -1}$ $\therefore$ $\boxed{C_2 = 2}$

For continues time system shown in the figure. Find zero input response solution, given initial condition $y(0)=2$, $\dot{y}(0)=6$



Solution:

$$e(t) = -8\,w(t) - 16\,y(t) + x(t)\cos 2t$$

$$\because\ e(t) = \frac{d^2y(t)}{dt^2}\ ,\quad w(t) = \frac{dy(t)}{dt}$$

$$\therefore\ \frac{d^2y(t)}{dt^2} = -8\frac{dy(t)}{dt} - 16\,y(t) + x(t)\cos(2t)$$

$$\frac{d^2y(t)}{dt^2} + 8\frac{dy(t)}{dt} + 16\,y(t) = x(t)\cos(2t)$$

$\because\ \frac{d}{dt} = D$ & ZI "zero input $x(t)=0$"

$$(D^2 + 8D + 16)\,y(t) = 0$$

$$(\lambda+4)(\lambda+4) = 0 \quad \therefore \lambda_1 = -4,\ \lambda_2 = -4$$

$$y_0(t) = C_1 e^{-4t} + C_2\,t\,e^{-4t}$$

from initial conditions $y(0)=2$, $\dot{y}(0)=6$

$$y_0(0) = 2 = C_1 + 0 \Rightarrow \boxed{C_1 = 2}$$

$$\dot{y}_0(t) = -4C_1e^{-4t} + C_2e^{-4t} - 4C_2te^{-4t}$$

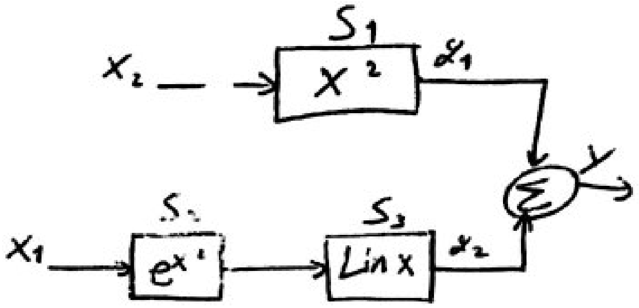
$$\dot{y}_0(0) = 6 = -4C_1 + C_2 \Rightarrow \boxed{C_2 = 14}$$

$$y_0(t) = 2e^{-4t} + 14te^{-4t} = e^{-4t}(2+14t)$$

$$\boxed{y_0(t) = e^{-4t}(2+14t)}$$

EX: a given system S_1, S_2 and S_3 are connec... own in the figure below, consider

$x_1 = \cos 2\pi t + \sin 2\pi t$. $x_2 = \sin 2\pi t$



1) check linearity for S_1 and S_2 .

2) check if S_2 ZIZO or BIBO, prove your answer.

3) Find system output equation 'y'.

4) IS S_1 is invertible system

Solution:

a) for S_1 when $x = x_1$ $y_1 = x_1^2$

when $x = x_2$ $y_2 = x_2^2$

$y_1 + y_2 = x_1^2 + x_2^2$

when $x = x_3 = x_1 + x_2$ $y_3 = (x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2$

$y_3 \neq y_1 + y_2$

$\therefore$ S_1 it isn't linear « non linear »

for S_2 when $x = x_1$ $y_1 = e^{x_1^2}$

when $x = x_2$ $y_2 = e^{x_2^2}$

$y_1 + y_2 = e^{x_1^2} + e^{x_2^2}$

when $x = x_3 = x_1 + x_2$ $y_3 = e^{(x_1+x_2)^2}$

$y_3 \neq y_1 + y_2$

$\therefore$ S_2 is not linear system

$x \longrightarrow \boxed{e^{x^2}} \longrightarrow y$

$x = 0 \Rightarrow y = 1$

S_2 is not zizo system

System S_2 is BIBO

because input $|x_1| < B < \infty$ B is finite value

and output is Bounded because $|y| < A$

where A is finit value

c) $y = y_1 + y_2$

$y_1 = \sin^2 2\pi t$

$y_2 = \ln e^{x_1^2} = x_1^2 \ln e = x_1^2$

$= \cos^2 2\pi t + 2\cos 2\pi t \sin 2\pi t + \sin^2 2\pi t$

$= 1 + 2\cos 2\pi t \sin 2\pi t$

$y = \sin^2 2\pi t + 1 + 2\cos 2\pi t \sin 2\pi t$

d) $S_1^{-1} = \pm\sqrt{x}$

System S_1 is non-invertible system because two input gives the same output

Ex (r.) Conside impulse response for a
$h(t) = e^{-t} u(t)$. Find the output of the syst
input signal is given by

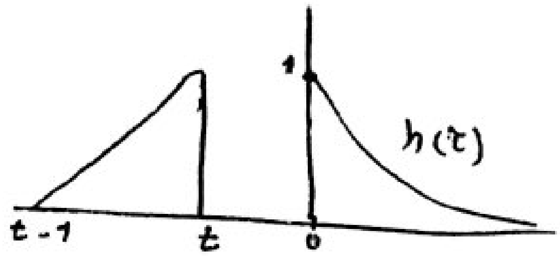
$$x(t) = \begin{cases} 1 - t & 0 \leq t \leq 1 \\ 0 & \text{other wise} \end{cases}$$

Solution

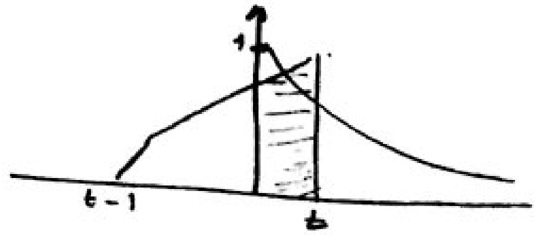
$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)$$

When $t < 0$

$y(t) = 0$



When $t > 0$
$t - 1 < 0 \Rightarrow 0 < t \leq 1$



$$\int_0^t (1 - (t-\tau))\, e^{-\tau}\, d\tau$$

$$y(t) = \int_0^t (1 - t + \tau)\, e^{-\tau}\, d\tau$$

$$= (1-t)\int_0^t e^{-\tau} d\tau + \underset{u}{\tau}\, \underbrace{e^{-\tau} d\tau}_{dv}$$

$u = \tau \qquad dv = e^{-\tau} d\tau$

$du = d\tau \qquad v = -e^{-\tau}$

$uv - \int v\, du$

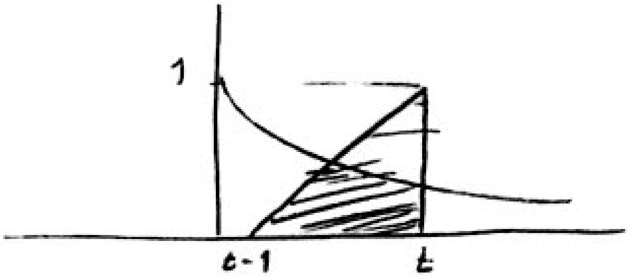
$$= (1-t)(-)\, e^{-\tau}\Big|_0^t + (-)e^{-\tau}\tau\Big|_0^t + \int_0^t e^{-\tau} d\tau$$

$$= (1-t)(-e^{-t} + 1) - e^{-t}t - e^{-\tau}\Big|_0^t$$

$$= -e^{-t} + 1 + te^{-t} - t - e^{-t}t - e^{-t} + 1$$

$$= 2 - t - 2e^{-t}$$

When $t-1>0 \Rightarrow t>1$



$$y(t) = \int (1-t+\tau)e^{-\tau}d\tau$$

$$= (1-t)(-)e^{-\tau}\Big|_{t-1}^{t} - e^{-\tau}\tau\Big|_{t-1}^{t} - e^{-\tau}\Big|_{t-1}^{t}$$

$$= (1-t)\left(-e^{-t}+e^{(t-1)}\right) - \left(e^{-t}t - e^{-(t-1)}(t-1)\right) - \left(e^{-t} - e^{-(t-1)}\right)$$

$$= -e^{-t} \cancel{+e^{-(t-1)}} \cancel{+te^{-t}} \cancel{-te^{-(t-1)}} \cancel{-e^{-t}t} \cancel{+te^{-(t-1)}} \cancel{-e^{-(t-1)}} - e^{-t} + e^{(t-1)}$$

$$= e^{-(t-1)} - 2e^{-t}$$

$$\therefore y(t) = \begin{cases} 0 & t<0 \\ 2-t-2e^{-t} & 0 \le t \le 1 \\ e^{-(t-1)} - 2e^{-t} & t>1 \end{cases}$$

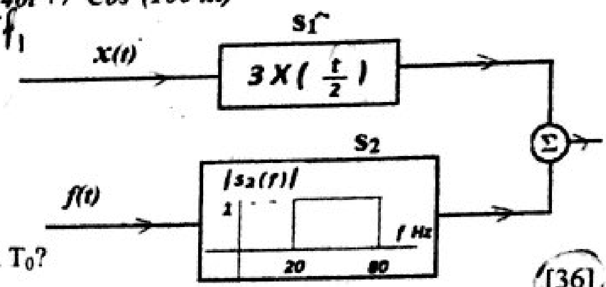
8

TRIPOLI UNIVERSITY
FACULTY OF ENGINEERING
Electrical and Electronic Engineering Department

Subject: Signals and Systems — Course: EE302 — Date: 12-06-2014
Spring 2014 — Final Exam: Problems [Marks: 80%] — Time: 125 minutes

Q1 Consider systems S_1 & S_2 interconnected as shown in the figure below, S_2 is band bass filter with unity amplitude , where :-

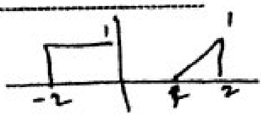
$$x(t) = \begin{cases} t-1 & 1 \le t \le 2 \\ 1 & -2 \le t \le 0 \\ 0 & \text{otherwise} \end{cases} \quad , \quad f(t) = 3 + \sin 40t + 7\,\cos^2(100\,\pi t)$$



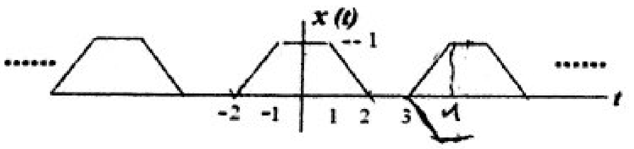
1- find and Sketch $x(t)$ even and odd parts
2- Sketch S_1 signal output
3- Find S_1 output energy
4- Dose S_1 time variant or invariant, and why?
5- Find output of S_2 , is it periodic or not, if yes find T_0?
6- Find over all system output power

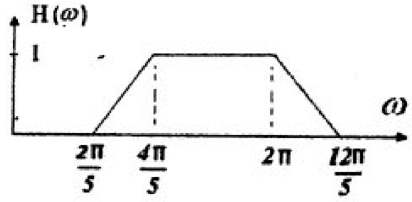
[36]

Q2 For a given periodic signal x(t) shown below is inserted to the BPF " $H(\omega)$ " given below



1) Find x (t) Exponential Fourier Series formula
2) Sketch X (t) spectrum, phase and amplitude for $n = 1 \longrightarrow 6$
3) Considering practical BPF cutoff frequency 3 dB below maximum value. Find system " $H(\omega)$ " output power

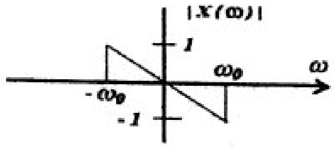


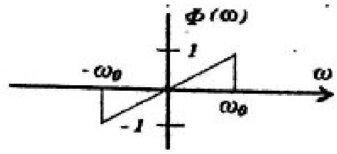


[28]

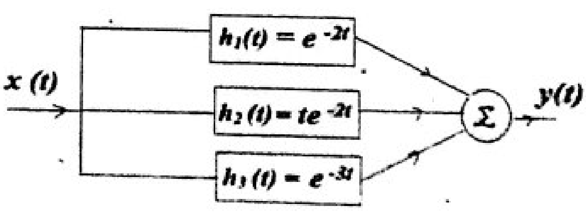
Q3 -

A- For a given signal $X(\omega)$ shown in the *Figure*
1) Find x (t)
2) Find signal energy
3) If X(t) modulated with a carrier $10\,\cos 5\omega_0 t$, sketch spectrum after modulation





B- Using Laplace transform for three parallel systems interconnection shown below, find:-
1- Over all system transfer function $H(s)$
2- Y(s). if $X(t) = t^2 e^{9t}$
3- System stability, and why ?



[36]

$\frac{4}{T_0}\left[\frac{1}{(n\omega_0)^2}\left\{\cos n\omega_0 t + n\omega_0 t \,\sin n\omega_0 t\right\}\right]_{-2}^{-1}$

$-\frac{2}{n\omega_0}\left[\sin n\omega_0 t\right]_{-2}^{-1} + \frac{1}{n\omega_0}\left[\sin n\omega_0 t\right]_{-1}^{0}$

$\int e^{-2t} e^{-st}\, dt \quad -\frac{1}{(s+2)} e^{-(s+2)t}$

$\int x e^{ax} dx = \frac{1}{a^2}(ax-1)e^{x}$

Good luck R.E

$$X(\omega) \begin{cases} -\frac{\omega}{\omega_0} e^{j\frac{1}{\omega_0}\omega} & -\omega_0 < \omega < \omega_0 \end{cases}$$

$$x(t) = \frac{-1}{2\pi\omega_0} \int_{-\omega_0}^{\omega_0} \omega e^{j\frac{\omega}{\omega_0}} \cdot e^{j\omega t} \, d\omega$$

$$= \frac{-1}{2\pi\omega_0} \int_{-\omega_0}^{\omega_0} \omega \, e^{j[\frac{1}{\omega_0}+t]\omega} \, d\omega \qquad \left(x e^{ax} = \left(\frac{x}{a} - \frac{1}{a^2}\right) e^{ax} \right)$$

$$= \frac{-1}{2\pi\omega_0} \left[\left(\frac{\omega}{j[\frac{1}{\omega_0}+t]} + \frac{1}{[\frac{1}{\omega_0}+t]^2} \right) e^{j[\frac{1}{\omega_0}+t]} \right]_{-\omega_0}^{+\omega_0}$$

Energy of $x(\omega) = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} |x(\omega)|^2 \, d\omega$

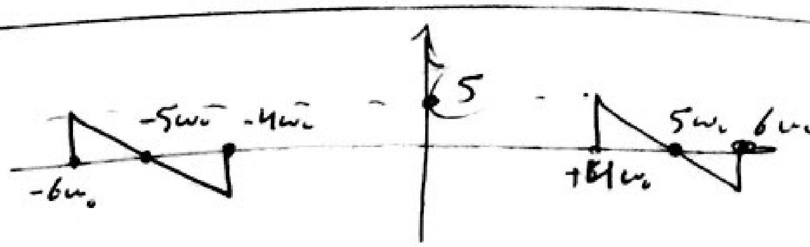
$$= \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} \left| \frac{-\omega}{\omega_0} \right|^2 d\omega$$

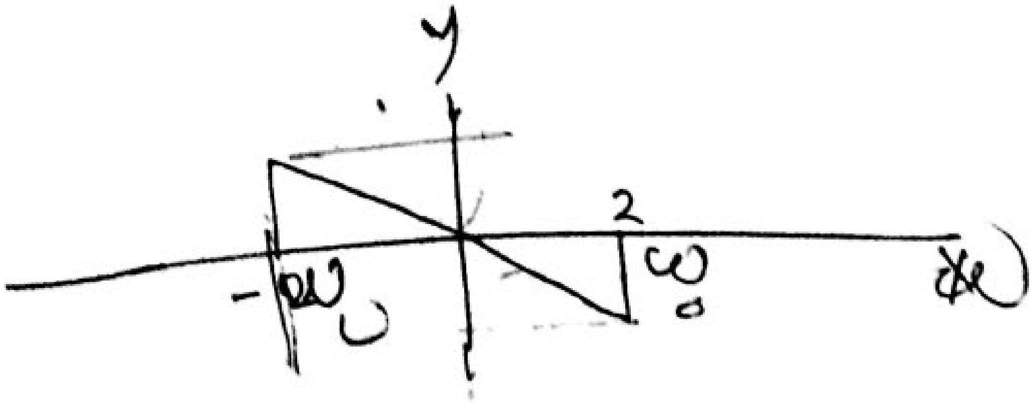
$$= \frac{1}{2\pi\omega_0^2} \int_{-\omega_0}^{\omega_0} \omega^2 \, d\omega$$

$$= \frac{1}{2\pi\omega_0^2} \left(\frac{\omega^3}{3} \Big|_{-\omega_0}^{\omega_0} \right) =$$

---

modulation





$$y = \frac{-1}{2\omega_0}\omega$$

UNIVERSITY OF TRIPOLI

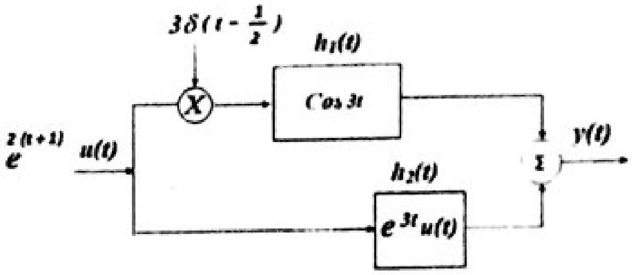
FACULTY OF ENGINEERING

Electrical and Electronic Engineering Department

Subject: Signals and Systems | Course Code: EE302 | Date: 12-05-2016

Spring 2016 | Marks: 20% | Time: 75 minutes

Note: for all question write steps of solution, indicate used tables in your answer if any, any direct result will not be considered. All sketches must be labelled carefully. Any multiple answers will not be considered



Q1 [16: 3, 3, 8]

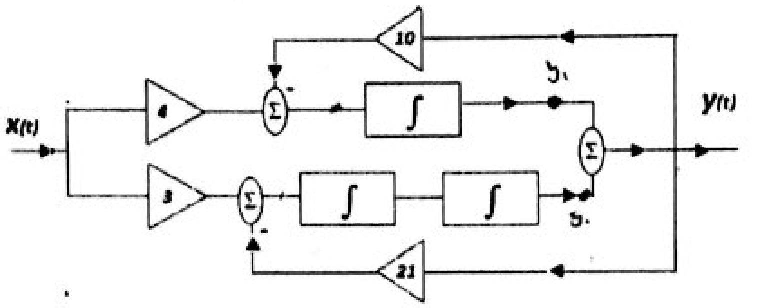
1- For $h_1(t)$, Check linearity, prove your answer

2- For $h_2(t)$ Check system stability, prove your answer

3- Find $y(t)$

Q2 [12] For LTI system shown in the figure below, find Zero Input response solution, given initial condition :-

$$y(0) = 0, \ \dot{y}(0) = 2$$

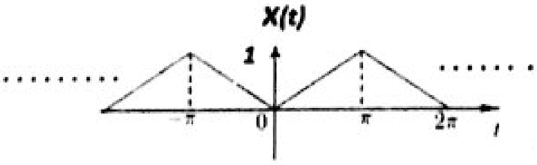


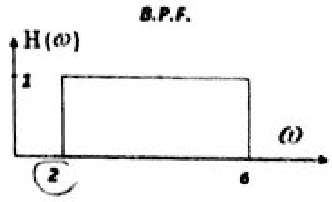
$\int y = t$

Q3 [18: 6, 6, 6]

For a given periodic signal $x(t)$ is inserted to the B.P.F " $H(\omega)$ " given below

1) Find x (t) T. F. S.C. formula
2) Sketch x (t) spectrum , phase and amplitude up to fifth component
3) Find B.P.F output power





[15]

$$\int t \sin at \ dt = \frac{1}{a^2}(\sin at - nt \cos at) + K$$

$$\int t \cos at \ dt = \frac{1}{a^2}(\cos at + nt \sin at) + K$$

Even symm :-

$\boxed{b_n = 0}$

$$a_0 = \frac{2}{T_0}\int_0^{T/2} x(t)\,dt$$

$$a_n = \frac{4}{T_0}\int_0^{T/2} x(t)\cos(n\omega_0 t)\,dt$$

Recall :> $\int x \cos ax = \frac{1}{a^2}[\cos ax + ax \sin ax]$

Good luck

$u = n\omega_0 t$

$du = n\omega_0 dt$

$\frac{u}{(n\omega_0)^2} \cos u \, dt$

(1)

$h_1(t) = t_1 \qquad h_1(t) = \cos 3t_1 .$

$h_2(t) = t_2 \qquad h_2(t) = \cos 3t_2$

$h_3(t) = t_3 \qquad h_3(t_3) = \cos 3t_3 \qquad h_3(t_3) = h_1(t_1) + h_2(t_2)$

$h_1(t_1) + h_2(t_2) = \cos 3t_1 + \cos 3t_2 \longrightarrow (1) \qquad t_3 = t_1 + t_2$

$h_1(t_1) + h_1(t_2) = \cos 3(t_1 + t_2) \longrightarrow (2)$

$(1) \neq (2)$

non linear #

(2) system is stable if ⇒

$|x(t)| < \beta < \infty \qquad t < \infty$

$e^{3\beta}$ and $\beta < \infty$

$e^{3\beta} = \infty$ only if $\beta = \infty$

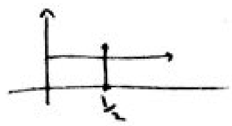
$\therefore e^{3t}$ is stable System. (BIBO)

(3)

$y(t) = y_1(t) + y_2(t)$

$y_1(t) = x_1(t) * h_1(t)$

$x_1(t) = e^{2(t+1)} u(t) \cdot 3\delta(t - \frac{1}{2})$



$= 3e^{3} \cdot \delta(t - \frac{1}{2})$

$y_1(t) = \int_{-\infty}^{\infty} 3e^{3} \delta(\tau - \frac{1}{2}) \cdot \cos 3(t - \tau) \, d\tau$

$$\boxed{y_1(t) = 3e^{3} \cos(3t - \frac{3}{2})}$$

$y_2(t) = \int_{-\infty}^{\infty} e^{2(\tau+1)} e^{3(t-\tau)} u(\tau) u(t-\tau) \, d\tau$

$= \int_0^t e^{2\tau} e^{2} e^{3t} e^{-3\tau} \, d\tau = e^{2+3t} \int_0^t e^{-\tau} \, d\tau = e^{2+3t} \left[-e^{-\tau} \right]_0^t$

$= e^{2+3t} \left[-e^{-t} - (-e^{0}) \right]$

$$y(t) = e^{2+3t} \left[-e^{-t} + 1 \right] = \boxed{\left(e^{2+3t} - e^{2+2t} \right) u(t)}$$

Q_2

$$y_1 = D^{-1}(4x - 10y)$$

$$y_2 = D^{-2}(3x - 21y)$$

$$y(t) = D^{-2}(3x - 21y) + D^{-1}(4x - 10y)$$

$$D^2 y(t) = 3x - 21y + D(4x - 10y)$$

$$D^2 y(t) + 10yD + 21y = \cancel{D4x}^{\,0} + 3x$$

$$\lambda^2 + 10\lambda + 21 = 0$$

$$(\lambda_1 + 3)(\lambda_2 + 7) = 0 \qquad \lambda_1 = -3 \quad \lambda_2 = -7$$

$$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

$$y(t) = C_1 e^{-3t} + C_2 e^{-7t}$$

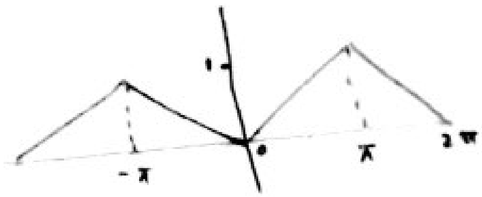
$$y'(t) = -3C_1 e^{-3t} - 7C_2 e^{-7t}$$

$$y(0) = \boxed{C_1 + C_2 = 0}$$

$$y'(0) = \boxed{-3C_1 - 7C_2 = 2}$$

$$\boxed{y(t) = \frac{1}{2} e^{-3t} - \frac{1}{2} e^{-7t}}$$

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$$



$$a_0 = \frac{1}{2\pi}\left[-\int_{-\pi}^{0}\frac{t}{\pi}\,dt + \int_0^{\pi}\frac{t}{\pi}\,dt\right]$$

$$= \frac{1}{2\pi}\left[\left[-\frac{t^2}{2\pi}\right]_{-\pi}^{0} + \left[\frac{t^2}{2\pi}\right]_0^{\pi}\right]$$

$$= \frac{1}{2\pi}\left(\left[0 - \frac{-(-\pi)^2}{2\pi}\right] + \left[\frac{\pi^2}{2\pi} - 0\right]\right)$$

$$= \frac{1}{2\pi}\left(\left[\frac{\pi^2}{2\pi}\right] + \left[\frac{\pi^2}{2\pi}\right]\right) = \frac{1}{2\pi}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = \frac{1(\pi}{2\pi} = \boxed{\frac{1}{2}}$$

$$a_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cos n\omega_0 t\,dt \qquad T_0 = 2\pi \quad \omega_0 = \frac{2\pi}{2\pi} = 1$$

$$a_n = \frac{2}{2\pi}\left[-\int_{-\pi}^{0}\frac{t}{\pi}\cos n\omega_0 t\,dt + \int_0^{\pi}\frac{t}{\pi}\cos n\omega_0 t\,dt\right]$$

$$a_n = \frac{1}{\pi^2}\left[\left[-\frac{1}{(n\omega_0)^2}\left(\cos n\omega_0 t + n\omega_0 t \sin n\omega_0 t\right)\right]_{-\pi}^{0} + \left[\frac{1}{(n\omega_0)^2}\left(\cos n\omega_0 t + n\omega_0 t \sin n\omega_0 t\right)\right]_0^{\pi}\right]$$

$$a_n = \frac{1}{\pi^2}\left(\left[\frac{-1}{(n\omega_0)^2}\left(\cancel{\cos 0}^{\,1} + \cancel{0\sin 0}^{\,0}\right) + \frac{1}{(n\omega_0)^2}\left(\cos -\pi n - \pi n \sin -\pi n\right)\right] + \right.$$

$$\left.\left[\frac{1}{(n\omega_0)^2}\left(\cos n\pi + n\pi \sin n\pi\right) - \frac{1}{(n\omega_0)^2}\left(\cancel{\cos 0}^{\,1} - \cancel{0\sin 0}^{\,0}\right)\right]\right)$$

$$a_n = \frac{1}{\pi^2}\left[\cancel{\frac{-1}{(n\omega_0)^2}} + \frac{1}{(n\omega_0)^2}\left(\cos \pi n - \cancel{\pi n \sin -\pi n}^{\,0}\right) + \frac{1}{(n\omega_0)^2}\left(\cos n\pi + \cancel{n\pi \sin n\pi}^{\,0}\right) - \cancel{\frac{1}{(n\omega_0)^2}}\right]$$

$$a_n = \frac{1}{(\pi n\omega_0)^2}\left[-2 + 2\cos \pi n\right] = \frac{2}{(\pi n \omega_0)^2}\left(\cos \pi n - 1\right)$$

$b_n = 0$ (even signal)

$$= \frac{2}{\pi^2 n^2}\left(\cos \pi n - 1\right)$$

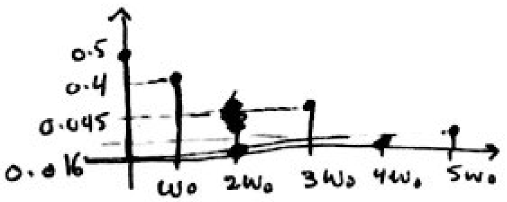
$$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)$$

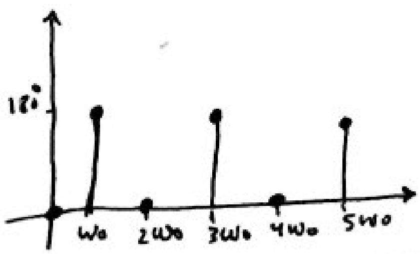
$$x(t) = \frac{1}{2} + \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left(\cos \pi n - 1\right)\cos nt$$

(2) $C_n = \sqrt{a_n^2 + b_n^2} = \sqrt{a_n^2 + 0} = a_n$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

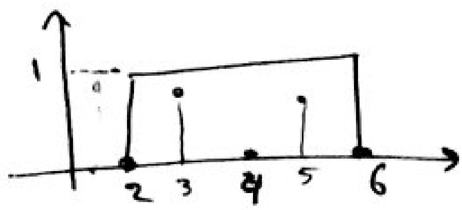
$$= \tan^{-1}\left(\frac{0}{a_n}\right) = \begin{cases} 0^\circ & a_n > 0 \\ 180^\circ & a_n < 0 \end{cases}$$





(3) Output power $= \frac{1}{2}\left[(0.045)^2 + (0.016)^2\right]$

$P = 1.1405\,mW$.



University Of Tripoli/Faculty of Engineering/EEE Department

EE302 Final Exam. / DATE: 26/7/2012

Q1):-

a) Express $z(t) = x(t) + y(t)$ as a single sinusoid:

$$x(t) = 3\cos(14\pi t - 0.2) \qquad y(t) = 2\cos(14\pi t + 0.10)$$ (2)

b) Find the time delay of the following cosine function with respect to the reference cosine function. (2)

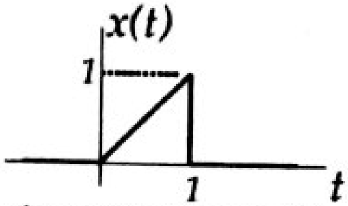
$$v(t) = 20\cos(400\pi t + 1.2)$$

$\frac{1.2}{400\pi}$, $t_d = \frac{\phi}{\omega}$

c) Evaluate the following integrals.

$$i) \int_0^3 e^{(t-2)}\,\delta(2t-4)\,dt. \qquad ii) \int_2^7 (t+1)\,\delta(t-1)\,dt.$$ (2) Zero

d) A sketch of a function x(t) is given below. Sketch (and label) the function $y(t) = 3x(4t-2)$. (2



e) Determine the power and rms. value for each of the following signals

$$i)\ 5 + 10\cos\left(100t + \frac{\pi}{3}\right) \qquad ii)\ 10e^{j100t}$$ (2)

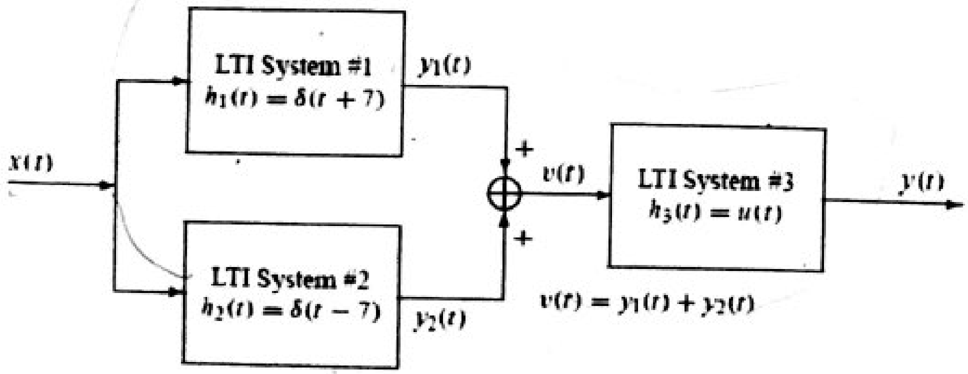
DC AC

Q2):-

a) Determine the unit impulse response $h(t)$ for a system specified by the equation

$$(D^2 + 3D + 2)y(t) = Dx(t)$$ (2.5)

b) What is the impulse response of the overall LTI system (i.e., from $x(t)$ to $y(t)$) ? Give your answer both as an equation and as a carefully labeled sketch. (2.5)



c) Suppose that the frequency response of a continuous time LTI system is $H(j\omega) = \frac{4-\omega^2}{1+j\omega}$ (5)

And the input is $x(t) = 4\cos(t) + \cos(2t)$. *determine the output of the system* $y(t)$.

Q3):-

a) The Trigonometric Fourier spectra of a certain periodic signal x(t) are shown in Fig.1

1) Sketch the exponential Fourier spectra. (2)
2) Verify your resulting analytically. (3)

Q3. a) If $y(t) = 2\cos^2(2t) + 4\sin\left(\frac{3}{2}t + 45^\circ\right) + e^{-j3t}$.

i. Find the exponential Fourier coefficient for y(t).
ii. Plot the line spectra for y(w).
iii. Find RMS value for the function y(t)

$\boxed{i}$ using $\cos 2t = 1 + \cos^2 t$

$$y(t) = 1 + \cos(4t) + 4\sin\left(\frac{3t}{2} + 45\right) + e^{-j3t}$$

from Euler's Formula

$$y(t) = 1 + \frac{1}{2}\left[e^{j4t} + e^{-j4t}\right] + \frac{2}{j}\left[e^{j3\frac{t}{2}} - e^{-j3\frac{t}{2}}\right] + e^{-j3t}$$

$$= 1 + \frac{1}{2}e^{j4t} + \frac{1}{2}e^{-j4t} - 2je^{j3\frac{t}{2}} + 2je^{-j3\frac{t}{2}} + e^{-j3t}$$

$$y(t) = \frac{1}{2}e^{-j4t} + e^{-j3t} + 2je^{-j\frac{3t}{2}} + 1 - 2je^{j\frac{3}{2}t} + \frac{1}{2}e^{j4t}$$ $\boxed{5}$

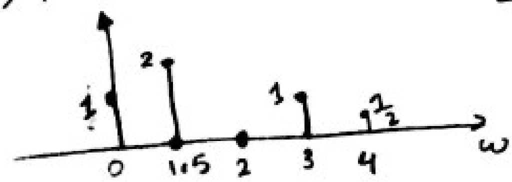
$\omega_0 = \frac{1}{2} \Longrightarrow T_0 = 4\pi$

$\therefore C_0 = 1$, $C_3\Big|_{\omega = 3\omega_0 = \frac{3}{2}} = -2j \Rightarrow C_{-3} = 2j$, $C_5\Big|_{\omega = 5\omega_0} = 0 = C_{-5}$

$C_2\Big|_{\text{at } \omega = 2\omega_0 = 1} = 0 = C_{-2}$, $C_4\Big|_{\omega = 4\omega_0 = 2} = 0 = C_{-4}$, $C_6\Big|_{\omega = 6\omega_0 = 3} = 0$ $C_{-6} = 1$

$C_8 = \frac{1}{2} \Rightarrow C_{-8} = \frac{1}{2}$ $\boxed{5}$

(ii) $y(t) = 1 + \cos(4t) + 4\sin\left(3\frac{t}{2} + 45\right) + \cos 3t - j\sin 3t$



(iii) $Rms = \sqrt{P} =$ $\boxed{5}$

$$P = C_0^2 + 2C_3^2 + 2C_6^2 + 2C_8^2 = 1 + 2\left[(2)^2 + (1)^2 + \left(\tfrac{1}{2}\right)^2\right]$$

$$= 1 + 2\left[4 + 1 + \frac{1}{4}\right]$$

$$= 1 + 8 + 2 + \frac{1}{2} = 11.5$$

$rms = \sqrt{11.5}$

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Q5. a) A continuous-time system is given by the input/output differential equation

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{d^2x(t)}{dt^2} - 4\frac{dx(t)}{dt} - x(t)$$

$$y(0) = -2, \quad \dot{y}(0) = 1, \quad x(t) = u(t)$$

$\boxed{5}$

Compute the response y(t) for all $t \geq 0$

$$[s^2Y(s) - s\,y(0) - \dot{y}(0)] + 4[sY(s) - y(0)] + 3Y(s)$$

$$= 2s^2X(s) - 4sX(s) - X(s)$$

$$\therefore Y(s)[s^2+4s+3] = -2s-7+[2s^2-4s-1]X(s)$$

$$Y(s) = \frac{-2s-7+[2s^2-4s-1]}{s^2+4s+3}X(s)$$

$$Y(s) = \frac{1}{s}$$

$$Y(s) = \frac{-2s-7+(2s^2-4s-1)}{(s+3)(s+1)} \cdot \frac{1}{s}$$

$$= \frac{-2s^2-7s+2s^2-4s-1}{s(s+3)(s+1)} = \frac{-11s-1}{s(s+3)(s+1)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+1}$$

$A = -\frac{1}{3}$

$B = \frac{16}{3}$

$C = -5$

$$y(t) = -\frac{1}{3} - 5e^{-t} + \frac{16}{3}e^{-3t}$$

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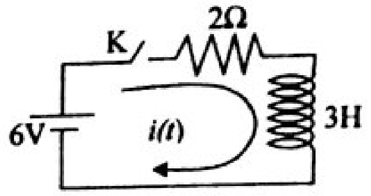
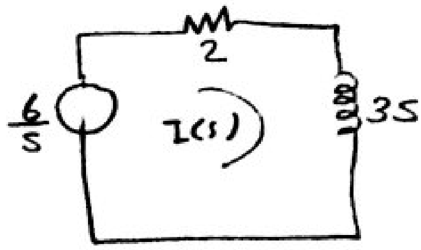
d) For the *RL* circuit shown in the Figure, if the switch K closed at $t = 0$, and the inductor is initially unfluxed $i_L(0^-) = 0$, find the current $i(t)$ for $t > 0$ using Laplace transform.

$\boxed{5}$



$$3\frac{di}{dt} + 2\,i(t) = 6\,u(t)$$

$i(0^-) = 0$



$$(3S+2)\,I(s) = \frac{6}{S}$$

$$I(s) = \frac{2}{S(S+\frac{2}{3})} = \frac{A}{S} + \frac{B}{S+1.5}$$

$$A = S\,I(s)\Big|_{S=0} = 3 \quad , B = (S+\tfrac{2}{3})\,I(s)\Big|_{S=-\frac{2}{3}} = -3$$

$$I(s) = \frac{3}{S} - \frac{3}{S+2/3} \Rightarrow i(t) = 3\left[1 - e^{-2/3t}\right]u(t)$$

GOOD LUCK FOR EVERY BODY

$\cancel{3}$

$$6 - R\,i(t) - L\frac{di}{dt} = 0$$

$$\frac{6}{S} = R\,I(s) + L\left[S\,I(s) - I(0^-)\right]$$

$a\cos\omega_0 t + b\sin\omega_0 t = c\cos(\omega_0 t+\theta)$

EE 302 Final Exam. solution

$\boxed{c=\sqrt{a^2+b^2}}$ $\boxed{\theta=\tan^{-1}(b/a)}$

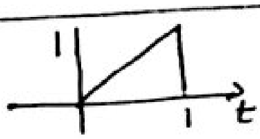
Q1) (a) $z(t)=4.946\cos(14\pi t-0.08)$ α [2]

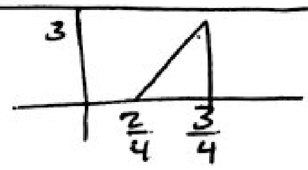
(b) $t_d=-\left(\frac{\phi}{\omega}\right)=-\left(\frac{1.2}{400\pi}\right)=-9.55\times10^{-5}s=-95.5\mu s$ [2]

(c) i) $\int_0^3 e^{(t-2)}\delta(2t-4)dt=\int_0^3 e^{(t-2)}\delta(2(t-2))dt=\frac{1}{2}e^{0}=\frac{1}{2}$ (1) [2]

$\boxed{\delta(2t)=\frac{\delta(t)}{a}}$

ii) $\int_2^7 (t+1)\delta(t-1)dt=0$ zero (1)

(d) 

$y(t)=3x(4t-2)\Rightarrow$ 

[2]

(e) i) $5+10\cos(100t+\frac{\pi}{3})\Rightarrow P=(5)^2+\frac{10^2}{2}=25+50=75$ (1) [2]

$\boxed{rms=\sqrt{75}}$

ii) $10e^{j100t}=10(\cos(100t)+j\sin 100t)\Rightarrow P=|D|^2=(10)^2=100$

$\boxed{rms=\sqrt{100}=10}$

(f) $Y(j\omega)=\frac{k_1}{1-j\omega}+\frac{k_2}{3+j\omega}+\frac{\pi}{3}\delta(\omega)+\frac{k_3}{j\omega}+\frac{k_4}{3+j\omega}$ Q3-(b)

$k_1=\frac{1}{3+j\omega}\Big|_{j\omega=1}=\frac{1}{2}$, $k_2=\frac{1}{1-j\omega}\Big|_{j\omega=-3}=\frac{1}{4}$ [2]

$k_3=\frac{1}{3+j\omega}\Big|_{j\omega=0}=\frac{1}{3}$, $k_4=\frac{1}{j\omega}\Big|_{j\omega=-3}=-\frac{1}{3}$

$Y(j\omega)=\frac{\frac{1}{2}}{(1-j\omega)}+\frac{\frac{1}{4}}{3+j\omega}+\frac{\pi}{3}\delta(\omega)+\frac{1}{3}\frac{1}{j\omega}-\frac{1}{3}\frac{1}{3+j\omega}$

$y(t)=\frac{1}{2}e^{t}u(-t)+\frac{1}{4}e^{-3t}u(t)+\frac{1}{3}u(t)-\frac{1}{3}e^{-3t}u(t)$

TRIPOLI UNIVERSITY
FACULTY OF ENGINEERING
Electrical and Electronic Engineering Department

Subject: Signals and Systems Date: 2/3/2013
Course No. : - EE302 Fall 2012 Final Exam. Time: 3 hours

Q1. For the giving signal $f(t)$;

a) Express the function $f(t)$ mathematically

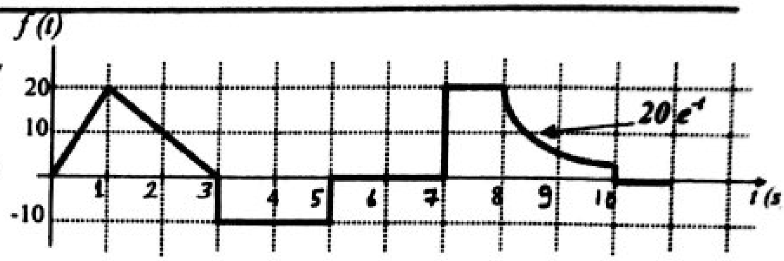
b) Sketch $f\left(\frac{2-t}{3}\right)$

c) Sketch or write down expression for the Following functions

i. $y_1(t) = f(t)u(3-t)$

ii. $y_2(t) = \int_1^3 f(t)\delta(2t-4)dt$

iii. $y_3(t) = \frac{dy_1(t)}{dt}$



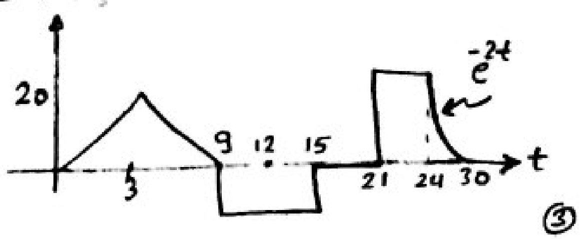
{6,6,2,2,4}

a) $f(t) = 20r(t) - 30r(t-1) + 10r(t-3) - 10u(t-3) + 10u(t-5)$

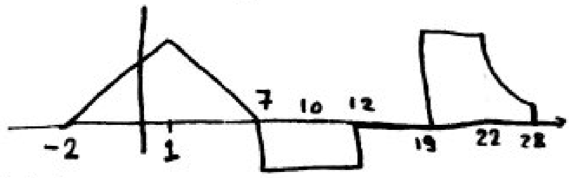
$+ 20u(t-7) - 20u(t-8) + 20e^{-(t-8)}[u(t-8) - u(t-10)]$ $\boxed{6}$

b) $f\left(\frac{2-t}{3}\right) \Rightarrow f(t) \xrightarrow{t \to \frac{t}{3}} f\left(\frac{t}{3}\right) \xrightarrow{t \to t+2} f\left(\frac{t+2}{3}\right) \xrightarrow{t \to -t} f\left(\frac{-t+2}{3}\right)$

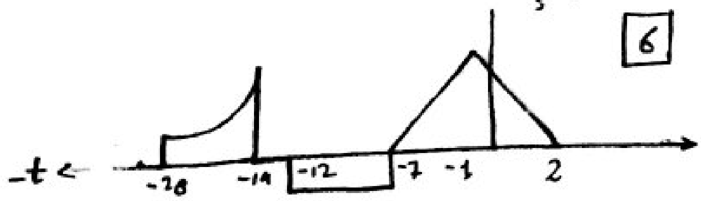
① $f\left(\frac{t}{3}\right)$

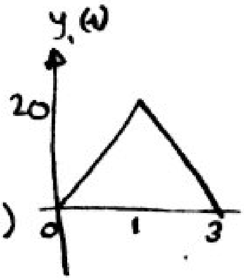


② $f\left(\frac{t+2}{3}\right)$



③ $f\left(\frac{-t+2}{3}\right)$ $\boxed{6}$

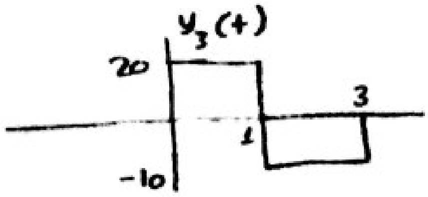




c) $\boxed{2}$ $\boxed{i}$ $y_1(t) = f(t)u(3-t) \Rightarrow 20r(t) - 30r(t-1) + 10r(t-3)$

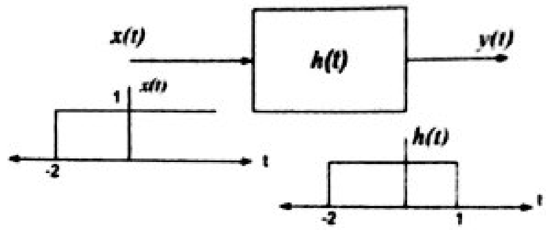
$\boxed{2}$ $\boxed{ii}$ $y_2(t) = \frac{1}{2}\int_1^3 f(t)\,\delta(t-2)dt = \frac{1}{2}f(2) = \frac{1}{2}\left[-10t+30\right]\Big|_{t=2} = \frac{1}{2}[10] = 5$

$\boxed{1}$ $\boxed{iii}$ $y_3(t) = \frac{dy_1(t)}{dt} = \frac{d}{dt}\left[20r(t) - 30r(t-1) + 10r(t-3)\right] = 20u(t) - 30u(t-1) + 10u(t$

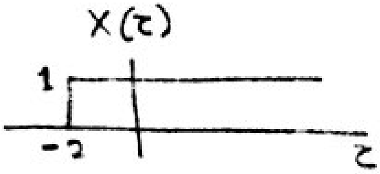


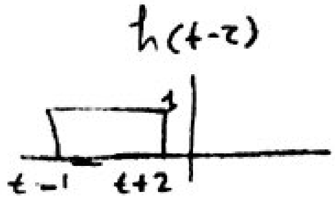
d) Find $y(t)$ for given $x(t)$ and $h(t)$

$\boxed{6}$

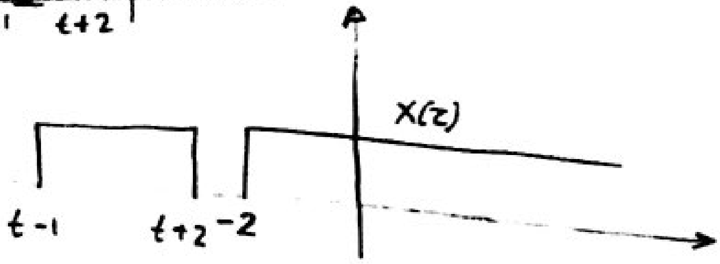


$$y(t) = \int_{-\infty}^{\infty} x(t)\, h(t-\tau)\, d\tau$$

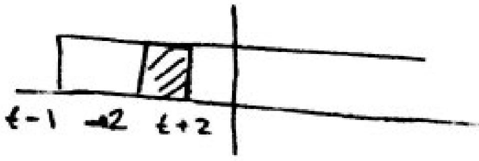




① $y(t) = 0 \qquad t \leq 4$

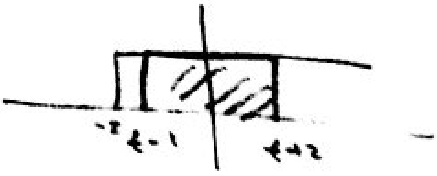


② $y(t) = \int_{-2}^{t+2} d\tau = t+4$

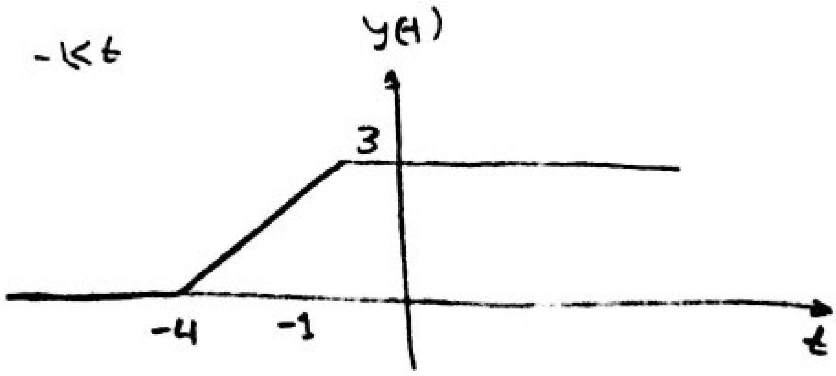


$y(t) = t+4 \qquad -4 \leq t \leq -1$

③ $y(t) = \int_{t-1}^{t+2} d\tau = 3 \qquad t \geq -1$



$$\therefore\ y(t) = \begin{cases} 0 & t \leq -4 \\ t+4 & -4 \leq t < -1 \\ 3 & -1 \leq t \end{cases}$$



c) Consider the interconnection of four LTI systems depicted below.

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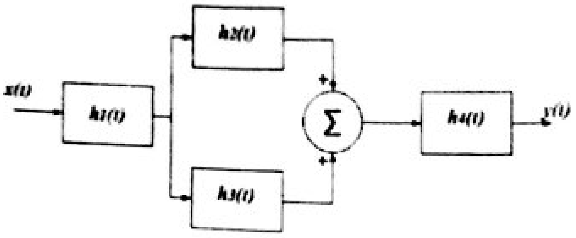


$\boxed{6}$

$h_1(t) = e^{-2t}u(t+1)$
$h_2(t) = u(t-3)$
$h_3(t) = 2\delta(t)$
$h_4(t) = 2\delta(t-1)$

i. Find the impulse response h(t) of the equivalent system.
ii. Which of the system $h_1(t), h_2(t), h_3(t), h_4(t)$ are stable? Show your work?

$\boxed{i}$ $h(t) = h_1(t) * [h_2(t) + h_3(t)] * h_4(t)$

$$= e^{-2t}u(t+1) * (u(t-3) + 2\delta(t)] * 2\delta(t-1)$$

$$= e^{-2t}u(t+1) * (2u(t-4) + 4\delta(t-1)]$$

$$h(t) = 2e^{-2t}u(t+1) * u(t-4) + 4e^{-2(t-1)}u(t)$$

$$2e^{-2t}u(t+1) * u(t-4) = \int_{-1}^{t-4} 3e^{-2\tau}d\tau = \frac{2}{2}\left[e^{2} - e^{-2(t-4)}\right]$$

$$\boxed{\therefore h(t) = \frac{3}{2}\left[e^{2} - e^{-2(t-4)}\right]u(t-3) + 2e^{-2(t-1)}u(t)} \#$$

$\boxed{ii}$ ① $\int_{-1}^{\infty} e^{-2t}dt = \frac{1}{2}e^{-2t}\Big|_{-1}^{\infty} = -\frac{1}{2}e^{2} \Rightarrow$ $\boxed{\text{stable}}$

② $\int_{3}^{\infty} dt = \infty \Rightarrow$ $\boxed{\text{not stable}}$

③ $\int_{-\infty}^{\infty} 2\delta(t)dt = 2 \Rightarrow$ $\boxed{\text{stable}}$

④ $\int_{-\infty}^{\infty} 3\delta(t-1) = 3 \Rightarrow$ $\boxed{\text{stable}}$

Q2. a) Determine whether the system $y(t) = K\frac{dx(t)}{dt} + 2$ is linear or not?

$y_1(t) = K x_1(t) + 2$

$y_2(t) = k x_2(t) + 2$

if $x_3(t) = x_1(t) + x_2(t) \Rightarrow y_3(t) = k(x_1(t) + x_2(t)) + 2$

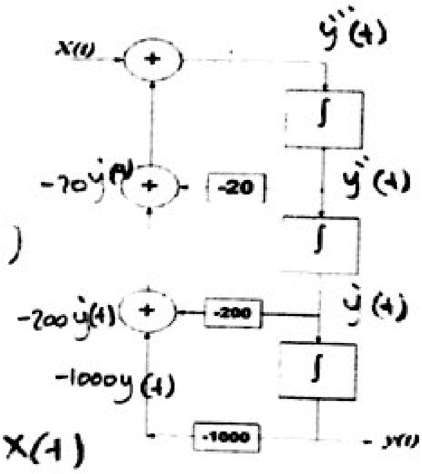
$y(t) = y_1(t) + y_2(t) = k(x_1(t) + x_2(t)) + 4$

$y_3(t) \neq y(t)$ [is not Linear]

$\boxed{4}$

b) For the system shown by the diagram below, determine the differential equation describing the system.

$\dddot{y}(t) = x(t) - 20\ddot{y}(t) - 200\dot{y}(t) - 1000y(t)$



$\dddot{y}(t) + 20\ddot{y}(t) + 200\dot{y}(t) + 1000y(t) = x(t)$

$\frac{d^3y(t)}{dt^3} + 20\frac{d^2y(t)}{dt^2} + 200\frac{dy(t)}{dt} + 1000y(t) = x(t)$

$\boxed{4}$